

Introduction to Languages for Scientific Computing

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Challenge: a difficult summation

Captain James Kirk of the starship Enterprise is faced with a difficult summation:

$$\sigma := (((.1 + .3) + .5) + .7) + .9).$$

The Enterprise is equipped with calculators which internally use the “Star Trek” arithmetic:

$$\beta = 2, \quad t = 4, \quad e_{\min} = -4, \quad e_{\max} = 4.$$

This arithmetic does not include subnormal numbers, NaNs, infinities, underflows, overflows and such.

Unfortunately, due to cosmic radiations, all calculators were affected by the “last-digit bug”: Input numbers could be represented only by mantissas ending with a 1.

As an example, the number $x = 2.0$ could only be represented either as $x_l = 1.111 \times 2^0$ or $x_u = 1.001 \times 2^1$; since $|x_l - 2| < |x_u - 2|$, 2 is represented as 1.111×2^0 .

Note: the last-digit bug manifests itself only when converting a real number to binary form. The addition is not affected, and binary numbers with a trailing zero are also accepted.

Last-digit bug

The objective of this challenge is to compute the error due to the last-digit bug in the computation of σ .

- **Step 1:** Compute σ_{ST} with a fully functional Star Trek arithmetic.
- **Step 2:** Compute σ_{Bug} with the Star Trek arithmetic, taking into account the last-digit bug.
- **Question A:** What is the relative error (E_{Bug}) for σ_{Bug} with respect to σ_{ST} ?
- **Question B:** What is the relative error (E_{ST}) for σ_{ST} with respect to the result in exact arithmetic?
- **Question C:** Compute the Machine Precision \mathbf{u} for the “good” Start Trek arithmetic, as the distance between 1 and the next floating point number. How does \mathbf{u} compare to E_{ST} ?

The first student who submits the right answer to all questions (A, B, and C) wins the challenge.

- Individual assignment
- Submit both the final answer and its derivation
- Submission by email to `pauldj@aices.rwth-aachen.de`
- Email's subject: `'LSC-17 Challenge1 <your last name>'`
- Accepted formats: plain text, pdf.
- Name your file `<your name>.txt` or `<your name>.pdf`
- **Deadline: The challenge is open until solved**