Tensor Computations: Efficiency Or Productivity?

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High Performance and Automatic Computing Part I

The HPC perspective

Disclaimer: Talk about awareness, not results

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- Target architecture? (parallel paradigm)

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 - \Rightarrow **Caveat**: Gains in the building blocks... often lost at the higher levels

Examples (1/2) Genome-Wide Association Studies (GWAS)

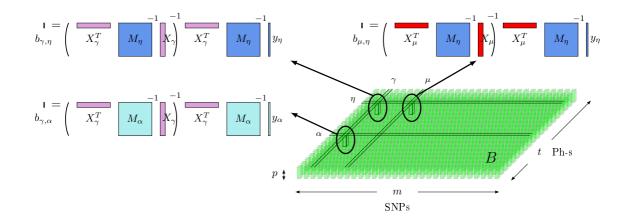
▶ What is it? Data correlation analysis. 2D grid of generalized least squares problems (GLS)

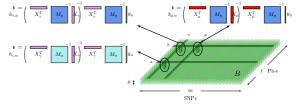
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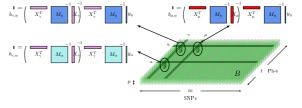
How is it related to tensors?

 $1D{\times}1D$ cartesian product of GLSs, 2D output = 4D data



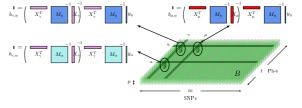


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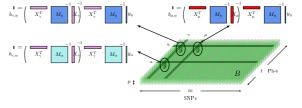
Library available: OmicABEL 🚺



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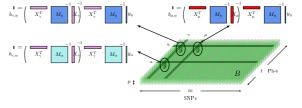


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- Interface: C vs. R
- Data management: data formats, overwriting, multiple files
- Data manipulation: imputation, filtering, selection
- Workflow not as fixed as first understood: M vs no-M, ...
- "The pre-processing is slower than the analysis"



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- "The pre-processing is slower than the analysis"
- $\Rightarrow\,$ Performance is important, but not as much as we like to think

Examples (2/2)

High-Performance Tensor Kernels

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Tensor Transpositions

$$\mathcal{B}_{i_1i_2...i_N} \leftarrow \alpha \cdot \mathcal{A}_{\pi(i_1i_2...i_N)} + \beta \cdot \mathcal{B}_{i_1i_2...i_N}$$

Summations — linear summation over tensor transpositions

$$\begin{split} \mathcal{B}_{i_0i_1i_2} &\leftarrow 2\mathcal{A}_{i_0i_1i_2} - \mathcal{A}_{i_2i_1i_0} - \mathcal{A}_{i_0i_2i_1} \\ \mathcal{B}_{i_0i_1i_2} &\leftarrow 4\mathcal{A}_{i_0i_1i_2} - 2\mathcal{A}_{i_1i_0i_2} - 2\mathcal{A}_{i_2i_1i_0} + \mathcal{A}_{i_1i_2i_0} - 2\mathcal{A}_{i_0i_2i_1} + \mathcal{A}_{i_2i_0i_1} \\ \mathcal{B}_{i_0i_1i_2i_3} &\leftarrow 2\mathcal{A}_{i_0i_1i_2i_3} - \mathcal{A}_{i_2i_1i_0i_3} - \mathcal{A}_{i_0i_2i_1i_3} - \mathcal{A}_{i_0i_1i_3i_2} \end{split}$$

Tensor Contractions

$$\mathcal{C}_{\pi_{\mathcal{C}}(I_m \cup I_n)} \leftarrow \alpha \cdot \mathcal{A}_{\pi_{\mathcal{A}}(I_m \cup I_k)} \times \mathcal{B}_{\pi_{\mathcal{B}}(I_n \cup I_k)} + \beta \cdot \mathcal{C}_{\pi_{\mathcal{C}}(I_m \cup I_n)}$$

Examples (2/2)Paul Springer

High-Performance Tensor Kernels

Tensor Transpositions

TTC: A high-performance Compiler for Tensor Transpositions. ACM TOMS, 2017 Compiler: https://github.com/HPAC/TTC Library: https://github.com/HPAC/hptt

Summations — linear summation over tensor transpositions

Spin Summations: A High-Performance Perspective. ACM TOMS, 2019 Generator: https://github.com/springer13/spin-summations

Tensor Contractions

Design of a high-performance GEMM-like Tensor-Tensor Multiplication. ACM TOMS, 2018 Compiler: https://github.com/HPAC/tccg Library: https://github.com/springer13/tcl

"Wrong" level of abstraction for domain scientists

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- $\blacktriangleright Mismatch \rightarrow mapping problem$

2D case: "Right" level of abstraction

Generalized Least Squares	$b := (X^T M^{-1} X)^{-1} X^T M^{-1} y$	$n > m$; $M \in \mathbb{R}^{n \times n}$, SPD; $X \in \mathbb{R}^{n \times m}$; $y \in \mathbb{R}^{n \times 1}$
Signal Processing	$x := (A^{-T}B^{T}BA^{-1} + R^{T}LR)^{-1}A^{-T}B^{T}BA^{-1}y$	
Kalman Filter	$K_k := P_k^b H^T (H P_k^b H^T + R)^{-1}; \ x_k^a := x_k^b + K_k (z_k - H x_k^b); \ P_k^a := (I - K_K H) P_k^b$	
Ensemble Kalman Filter	$X^{a} := X^{b} + (B^{-1} + H^{T}R^{-1}H)^{-1}(Y - HX^{b})$	
Image Restoration	$x_k := (H^T H + \lambda \sigma^2 I_n)^{-1} (H^T y + \lambda \sigma^2 (v_{k-1} - u_{k-1}))$	
Rand. Matrix Inversion	$X_{k+1} := S(S^{T}AS)^{-1}S^{T} + (I_{n} - S(S^{T}AS)^{-1}S^{T}A)X_{k}(I_{n} - AS(S^{T}AS)^{-1}S^{T})$	
Stochastic Newton	$B_k := \frac{k}{k-1} B_{k-1} (I_n - A^T W_k ((k-1)I_l + W_k^T A B_{k-1} A^T W_k)^{-1} W_k^T A B_{k-1})$	
Optimization	$x_f := WA^T (AWA^T)^{-1} (b - Ax); x_o := W (A^T (AWA^T)^{-1} Ax - c)$	
Tikhonov Regularization	$x := (A^T A + \Gamma^T \Gamma)^{-1} A^T b$	$A \in \mathbb{R}^{n imes m}; \ \Gamma \in \mathbb{R}^{m imes m}; \ b \in \mathbb{R}^{n imes 1}$
Gen. Tikhonov Reg.	$x := (A^T P A + Q)^{-1} (A^T P b + Q x_0)$	$P \in \mathbb{R}^{n \times n}$, SSPD; $Q \in \mathbb{R}^{m \times m}$, SSPD; $x_0 \in \mathbb{R}^{m \times 1}$
LMMSE estimator	$K_{t+1} := C_t A^T (A C_t A^T + C_z)^{-1}; \ x_{t+1} := x_t + K_{t+1} (y - A x_t); \ C_{t+1} := (I - K_{t+1} A) C_t$	





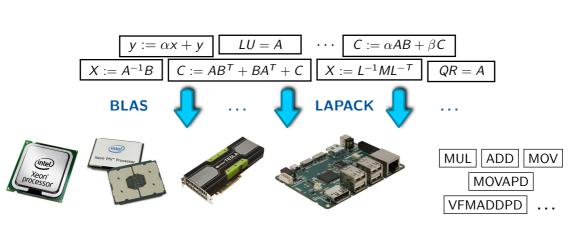
$$E := Q^{-1}U(I + U^T Q^{-1}U)^{-1}U^T$$
 ...

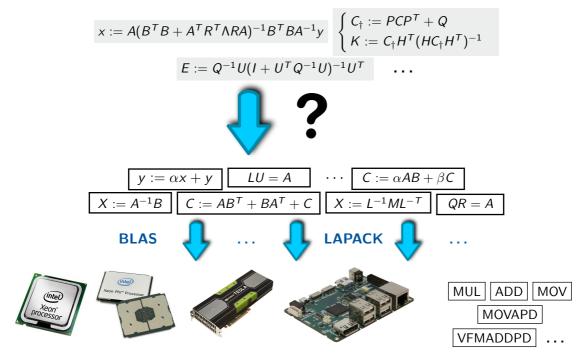
$$x := A(B^{\mathsf{T}}B + A^{\mathsf{T}}R^{\mathsf{T}}\Lambda RA)^{-1}B^{\mathsf{T}}BA^{-1}y \quad \begin{cases} C_{\dagger} := PCP^{\mathsf{T}} + Q \\ K := C_{\dagger}H^{\mathsf{T}}(HC_{\dagger}H^{\mathsf{T}})^{-1} \end{cases}$$

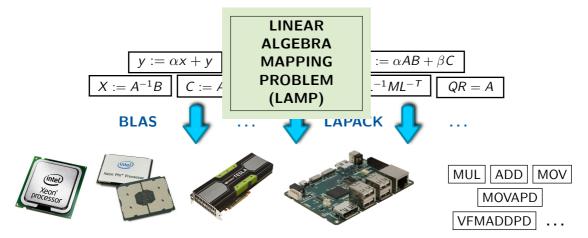
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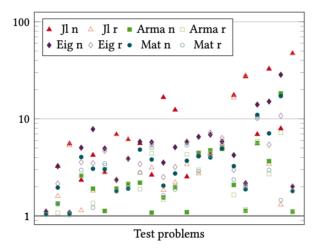
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LAMP: A problem often ignored

Linnea: A compiler for linear algebra – Henrik Barthels, Christos Psarras

Linnea's speedups



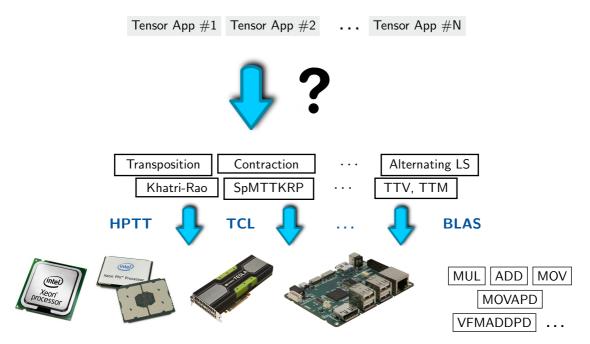
JI: Julia, Arma: Armadillo, Eig: Eigen, Mat: Matlab.

n/**r**: naive/recommended implementation

Tensor App #1 Tensor App #2 Tensor App #N







nD case: Exemplary applications

Coupled-Cluster methods

 $\tau_{ii}^{ab} = t_{ii}^{ab} + \frac{1}{2} P_b^a P_i^i t_i^a t_i^b$ $\tilde{F}_e^m = f_e^m + \sum v_{ef}^{mn} t_n^f,$ $ilde{F}_e^a = (1-\delta_{ae})f_e^a - \sum ilde{F}_e^m t_m^a - rac{1}{2}\sum v_{ef}^{mn}t_{mn}^{af} + \sum v_{ef}^{an}t_n^{f},$ $\tilde{F}_i^m = (1-\delta_{mi})f_i^m + \sum \tilde{F}_e^m t_i^e + \frac{1}{2}\sum v_{ef}^m t_{in}^{ef} + \sum v_{if}^{mn} t_n^f,$ $ilde{W}_{ei}^{mn} = v_{ei}^{mn} + \sum v_{ef}^{mn} t_i^f,$ $\tilde{W}_{ij}^{mn} = v_{ij}^{mn} + P_j^i \sum v_{ie}^{mn} t_j^e + \frac{1}{2} \sum v_{ef}^{mn} \tau_{ij}^{ef},$ $\tilde{W}_{ie}^{am} = v_{ie}^{am} - \sum \tilde{W}_{ei}^{mn} t_n^a + \sum_{e} v_{ef}^{ma} t_i^f + \frac{1}{2} \sum_{e} v_{ef}^{mn} t_i^{af},$ $ilde{W}^{am}_{ij} = v^{am}_{ij} + P^i_j \sum v^{am}_{ie} t^e_j + rac{1}{2} \sum v^{am}_{ef} au^{ef}_{ij},$ $z_i^a = f_i^a - \sum \tilde{F}_i^m t_m^a + \sum f_e^a t_i^e + \sum v_{ei}^m t_m^e + \sum v_{im}^{ae} \tilde{F}_e^m + \frac{1}{2} \sum$ $z_{ij}^{ab} = v_{ij}^{ab} + P_j^i \sum v_{ie}^{ab} t_j^e + P_b^a P_j^i \sum \tilde{W}_{ie}^{am} t_{mj}^{eb} - P_b^a \sum \tilde{W}_{ij}^{am} t_m^b + P_b^a P_b^a \sum v_{ij}^{am} t_m^b + P_b^a \sum v_$

credits to D. Matthews, E. Solomonik, J. Stanton, and J. Gauss

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Finite Element 3D diffusion operator

```
TE.BeginMultiKernelLaunch():
TE("T2 e i1 i2 k3 = B k3 i3 X e i1 i2 i3", T2, B, X);
TE("T1 e i1 k2 k3 = B k2 i2 T2 e i1 i2 k3", T1, B, T2);
TE("U1_e_k1_k2_k3 = G_k1_i1 T1_e_i1_k2_k3", U1, G, T1);
TE("T1 e i1 k2 k3 = G k2 i2 T2 e i1 i2 k3", T1, G, T2);
TE("U2_e_k1_k2_k3 = B_k1_i1 T1_e_i1_k2_k3", U2, B, T1);
TE("T2 e i1 i2 k3 = G k3 i3 X e i1 i2 i3", T2, G, X);
TE("T1 e i1 k2 k3 = B k2 i2 T2 e i1 i2 k3", T1, B, T2);
TE("U3 e k1 k2 k3 = B k1 i1 T1 e i1 k2 k3", U3, B, T1);
TE("Z m e k1 k2 k3 = U n e k1 k2 k3 D e m n k1 k2 k3", Z, U,
TE("T1 e i3 k1 k2 = B k3 i3 Z1 e k1 k2 k3", T1, B, Z1);
TE("T2 e i2 i3 k1 = B k2 i2 T1 e i3 k1 k2", T2, B, T1);
TE("Y e i1 i2 i3 = G k1 i1 T2 e i2 i3 k1", Y, G, T2);
TE("T1 e i3 k1_k2 = B_k3_i3 Z2_e_k1_k2_k3", T1, B, Z2);
TE("T2 e i2 i3 k1 = G k2 i2 T1 e i3 k1 k2", T2, G, T1);
TE("Y e i1 i2 i3 += B k1 i1 T2 e i2 i3 k1", Y, B, T2);
TE("T1 e i3 k1 k2 = G k3 i3 Z3 e k1 k2 k3", T1, G, Z3);
TE("T2 e i2 i3 k1 = B k2 i2 T1 e i3 k1 k2", T2, B, T1);
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TE.EndMultiKernelLaunch();
```

credits to A. Fisher - https://github.com/LLNL/acrotensor

credits to D. Matthews, E. Solomonik, J. Stanton, and J. Gauss

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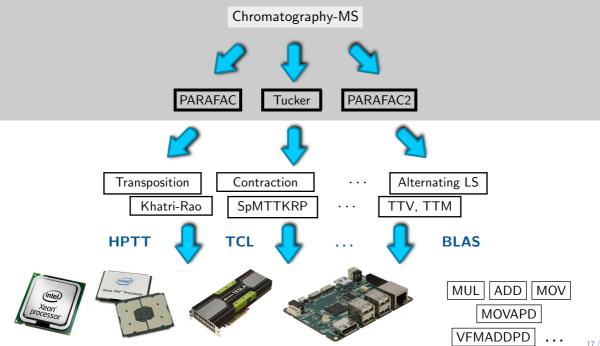
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Beware: It's challenging even for "simple" matrix computations!

Part II

The computational scientists' perspective

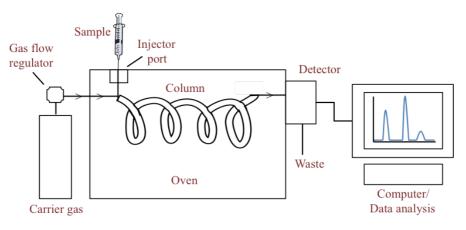
"The fastest FLOPS are those that are not executed." – Lars



Example application: Untargeted chemical profiling

Chromatography with mass spectrometry detection

Problem: Identify components in a sample

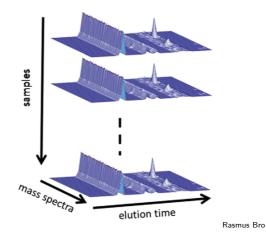


Jessica Torres – Bitesize Bio

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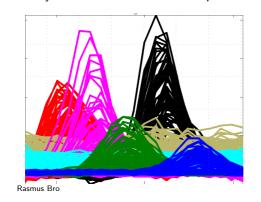
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▶ 3-way data: Mass-spectrum × elution time × sample

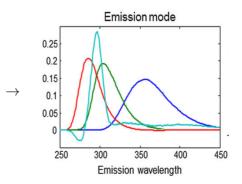


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 \blacktriangleright 3-way tensor \rightarrow Individual components



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 - Determine which of the components represent chemical information
 - Start over; add/change constraints, change model

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