Introduction to Scientific Computing Languages
Practice questions – Mathematica

Prof. **Paolo Bientinesi**

pauldj@aices.rwth-aachen.de

RWTH AACHEN UNIVERSITY

AICES

Deutsche Forschungsgemeinschaft DFG

Exercises (1/3)

1) Write a function that takes 5 reals $\{a, b, c\}, \{x_0, y_0\}$, computes the first 10.000 elements of the sequence

$$\begin{cases} x_{n+1} \leftarrow y_n - sign(x_n)\sqrt{|bx_n - c|} \\ y_{n+1} \leftarrow a - x_n \end{cases},$$

and plots them in the plane. Test the function with $\{\{.4, 1., 1.\}, \{0.0, 0.0\}\}$, $\{\{.4, 1., 1.\}, \{.2, .4\}\}$, and $\{\{1.4, 1.1, 2.2\}, \{.2, .99\}\}$.

2) Write a one liner that returns a 10-row $\times$ 3-column table that shows the frequency of digit i in the first 10^j digits of π, with $i = 0, \ldots, 9$, and $j = 1, \ldots, 5$.

Exercises (2/3)

3a) Let `l` be a list of pairs.

Write a function `noDup[l_]`, that returns `l` without any duplicates.

- `{x, y}` and `{x, y}` are duplicates.
- `{x, y}` and `{y, x}` are also duplicates.
- Do not alter the order of the entries.
- For duplicates, only the leftmost instance is kept.

3b) Write a function `noDupS[l_]`, which removes duplicates, and returns a sorted list of sorted pairs.

Exercises (3/3)

4) "Cycles"

Input: A permutation p of size n (a list containing the first n integers)

Output: The list of cycles in p

Goal: Write the function `cycles` that takes p and returns its cycles.

Example #1

Input = {2, 8, 4, 3, 5, 7, 6, 1}; Output = {{1, 2, 8}, {3, 4}, {5}, {6, 7}}

Explanation:
Start with the number 1; look at the entry in position 1, it is a 2; look at the entry in position 2, it is a 8; look at the entry in position 8, it is a 1; you returned to 1, the cycle {1,2,8} is closed. The next number that is not in a cycle is 3; look at the entry in position 3, it is a 4; look at the entry in position 4, it is a 3; the cycle {3,4} is closed. . . .

Example #2

Input = {4, 5, 2, 3, 1}; Output = {{1, 4, 3, 2, 5}}

Sequence

Definition
Let $\mathcal{S}_n$ be an integer, and $\#_k^{(n)}$ $(0 \leq k \leq 9)$ the number of occurrences of the digit k in $\mathcal{S}_n$. The sequence $\mathcal{S}$ is defined by the rule
$\mathcal{S}_{n+1} := \lll \#_0^{(n)}, 0, \#_1^{(n)}, 1, \ldots, \#_9^{(n)}, 9 \ggg$, for all the $\#_k^{(n)} > 0$.
$\lll$ ... $\ggg$ indicates the concatenation of the digits.

Examples
If $\mathcal{S}_n = 42$, then $\mathcal{S}_{n+1} = 1214$, and $\mathcal{S}_{n+2} = 211214$.
If $\mathcal{S}_n = 420000000000$, then $\mathcal{S}_{n+1} = 1001214$, $\mathcal{S}_{n+2} = 20311214$.

Computation
$\mathcal{S}$ might converge to a single number or to a loop (this is not important).
To study its evolution, use the following piece of code,
in which the function `iterRule[S_]` is user-defined:

```
FromDigits /@
   FixedPointList[iterRule, IntegerDigits@Sn, 50]
```

Goal
Define the function `iterRule[S_]`.

Plotting polynomials

Write the function `polyPlot[p_]`

Input

A polynomial p of unknown degree $n > 2$

Output

A 2d plot of p in the region of interest, suitably annotated

Goals

- Identify & highlight interesting features of p;
  these features determine the region of interest
- Add suitable annotations/labels
- Use `Manipulate` to slide an object along the polynomial

Possible features of interest:
zeros ($p(x) = 0$), maxes & mins ($p'(x) = 0$), saddles ($p''(x) = 0$), intersections with $y = x$ ($p(x) = x$), $p'(x) = \pm 1$, ...

Plotting polynomials: example (annotations missing)

polynomial:

```
p[x_] := x^7 - 2 x^4 + x + .5
```

invocation:

```
polyPlot[p]
```

or equivalently:

```
polyPlot[#^7 - 2 #^4 + # + .5&]
```