Introduction to Languages for Scientific Computing

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Floating Point Arithmetic

- **[Q1]** Consider the IEEE settings for single precision arithmetic:

$$\beta = 2, \quad t = 24, \quad e_{\min} = -125, \quad e_{\max} = 128$$

1. What is the smallest floating point number larger than 2?
2. What is the largest floating point number smaller than 8?
3. How many floating point numbers are in the interval [1/64, 1/32] ?
4. What is the distance between 65536 and the next floating point number?
5. What is the first integer that cannot be represented exactly?

- **[Q2]** Consider the following ternary arithmetic with normalization:

$$\beta = 3, \quad t = 3, \quad e_{\min} = -2, \quad e_{\max} = 3$$

  1. How is π represented? What is the representation error?
  2. What is the largest floating point number?
  3. What are the first 5 positive integers that cannot be represented exactly?

- **[Q3]** Consider the following binary arithmetic with normalization:

$$\beta = 2, \quad t = 4, \quad e_{\min} = -2, \quad e_{\max} = 4$$

  1. How is π represented? What is the representation error?
  2. What is the smallest absolute distance between two floating point numbers*?
  3. What is the smallest relative distance between two floating point numbers*?

*: the arithmetic is normalized. What if this is not the case?

- **[Q4]** Counting floating point numbers
  1. Let I_i be the interval $]2^i, 2^{i+1}[$, with $i = 0, \ldots, 15$.
  2. For each $i = 0, \ldots, 15$,
     let $\overleftarrow{s_i}$ and $\overrightarrow{s_i}$ be two **consecutive** single precision floating point numbers in I_i.
  3. Let d_i be the number of double precision numbers in the interval $]\overleftarrow{s_i}, \overrightarrow{s_i}[$.
  4. Compute $D = \sum_{i=0}^{15} d_i$.
  5. Is D representable exactly in single precision?
     If not, what are the absolute and relative representation errors?
  6. Is D representable exactly in double precision?
     If not, what are the absolute and relative representation errors?

Submission

- Individual assignment.
- Submit both the final answers and their derivation.
- Submission by email to `pauldj@aices.rwth-aachen.de`
- Email's subject: "`LSC-15 HW1 <your last name>`"
- Accepted formats: plain text or pdf.
- Name your file `<your name>.txt` or `<your name>.pdf`.
- Include your name in each attached file.
- **Deadline: Wednesday, November 4, 23.59pm.**