Introduction to Scientific Computing Languages
Practice questions – Mathematica

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1) Write a function that takes 5 reals $\{a, b, c\}, \{x_0, y_0\}$, computes the first 10.000 elements of the sequence

$$\begin{cases} x_{n+1} \leftarrow y_n - sign(x_n)\sqrt{|bx_n - c|} \\ y_{n+1} \leftarrow a - x_n \end{cases} ,$$

and plots them in the plane. Test the function with $\{\{.4, 1., 1.\}, \{0.0, 0.0\}\}$, $\{\{.4, 1., 1.\}, \{.2, .4\}\}$, and $\{\{1.4, 1.1, 2.2\}, \{.2, .99\}\}$.

2) Write a one liner that returns a 10-row $\times$ 3-column table that shows the frequency of digit i in the first 10^j digits of π, with $i = 0, \ldots, 9$, and $j = 1, \ldots, 5$.

3a) Let `l` be a list of pairs.

Write a function `noDup[l_]`, that returns `l` without any duplicates.

- `{x, y}` and `{x, y}` are duplicates.
- `{x, y}` and `{y, x}` are also duplicates.
- Do not alter the order of the entries.
- For duplicates, only the leftmost instance is kept.

3b) Write a function `noDupS[l_]`, which removes duplicates, and returns a sorted list of sorted pairs.

Sequence

Definition
Let $\mathcal{S}_n$ be an integer, and $\#_k^{(n)}$ $(0 \leq k \leq 9)$ the number of occurrences of the digit k in $\mathcal{S}_n$. The sequence $\mathcal{S}$ is defined by the rule
$\mathcal{S}_{n+1} := \lll \#_0^{(n)}, 0, \#_1^{(n)}, 1, \ldots, \#_9^{(n)}, 9 \ggg$, for all the $\#_k^{(n)} > 0$.
$\lll$ ... $\ggg$ indicates the concatenation of the digits.

Examples
If $\mathcal{S}_n = 42$, then $\mathcal{S}_{n+1} = 1214$, and $\mathcal{S}_{n+2} = 211214$.
If $\mathcal{S}_n = 420000000000$, then $\mathcal{S}_{n+1} = 1001214$, $\mathcal{S}_{n+2} = 20311214$.

Computation
$\mathcal{S}$ might converge to a single number or to a loop (this is not important).
To study its evolution, use the following piece of code,
in which the function `iterRule[S_]` is user-defined:

```
FromDigits /@
   FixedPointList[iterRule, IntegerDigits@Sn, 50]
```

Goal
Define the function `iterRule[S_]`.

Plotting polynomials

Write the function `polyPlot[p_]`

Input

A polynomial p of unknown degree $n > 2$

Output

A 2d plot of p in the region of interest, suitably annotated

Goals

- Identify & highlight interesting features of p;
  these features determine the region of interest
- Add suitable annotations/labels
- Use `Manipulate` to slide an object along the polynomial

Possible features of interest:
zeros ($p(x) = 0$), maxes & mins ($p'(x) = 0$), saddles ($p''(x) = 0$), intersections with $y = x$ ($p(x) = x$), $p'(x) = \pm 1$, ...

Plotting polynomials: example (annotations missing)

polynomial:

```
p[x_] := x^7 - 2 x^4 + x + .5
```

invocation:

```
polyPlot[p]
```

or equivalently:

```
polyPlot[#^7 - 2 #^4 + # + .5&]
```