Introduction to Scientific Computing Languages

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Challenge #2: `nMatrix, nMatrixRec, nMatrixMV`

1) Functions `nMatrix` and `nMatrixRec`

Input: an integer n

Output: the "nMatrix" N_n, as defined in the next slide.

2) Function `nMatrixMV`

Input: a vector v of size $n = 2 * t + 1$, where t is an integer

Output: the vectors y_{even} and y_{odd}, also of size n, resulting from the multiplication of the "even" and "odd" nMatrices (of suitable size) with the input vector v.
Mathematically: $y_{\text{even}} = N_{\text{even}} * v, \quad y_{\text{odd}} = N_{\text{odd}} * v.$

Challenge: Write the Matlab functions `nMatrix`, `nMatrixRec`, and `nMatrixMV`.

Goal #1 (`nMatrix`): Let the code speak for itself. Non-recursive definition.

Goal #2 (`nMatrixRec`): Let the code speak for itself. Recursive definition.

Goal #3 (`nMatrixMV`): Cleverness, not brute force. Do NOT form the nMatrices.

Definition of N_n

$$N_1 = [1] \quad N_2 = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 2 \end{bmatrix} \quad N_3 = \begin{bmatrix} 2 & 3 & -2 \\ 3 & 1 & 3 \\ -2 & 3 & 2 \end{bmatrix}$$

if $n > 3$ and even, $N_n :=$

$$\begin{bmatrix}
n & & & & 0 & & & & -n \\
& n-2 & & & n-1 & & & -(n-2) & \\
& & \ddots & & \vdots & & \iddots & & \\
& & & 2 & 3 & -2 & & & \\
0 & n-1 & \dots & 3 & 1 & 3 & \dots & n-1 & 0 \\
& & & -2 & 3 & 2 & & & \\
& & \iddots & & \vdots & & \ddots & & \\
& -(n-2) & & & n-1 & & & n-2 & \\
-n & & & & 0 & & & & n
\end{bmatrix}$$

if $n > 3$ and odd, $N_n :=$

$$\begin{bmatrix}
n-1 & & & & n & & & & -(n-1) \\
& n-3 & & & n-2 & & & -(n-3) & \\
& & \ddots & & \vdots & & \iddots & & \\
& & & 2 & 3 & -2 & & & \\
n & n-2 & \dots & 3 & 1 & 3 & \dots & n-2 & n \\
& & & -2 & 3 & 2 & & & \\
& & \iddots & & \vdots & & \ddots & & \\
& -(n-3) & & & n-2 & & & n-3 & \\
-(n-1) & & & & n & & & & n-1
\end{bmatrix}$$

Examples

$$N_4 = \begin{bmatrix} 4 & 0 & 0 & 0 & -4 \\ 0 & 2 & 3 & -2 & 0 \\ 0 & 3 & 1 & 3 & 0 \\ 0 & -2 & 3 & 2 & 0 \\ -4 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$$N_5 = \begin{bmatrix} 4 & 0 & 5 & 0 & -4 \\ 0 & 2 & 3 & -2 & 0 \\ 5 & 3 & 1 & 3 & 5 \\ 0 & -2 & 3 & 2 & 0 \\ -4 & 0 & 5 & 0 & 4 \end{bmatrix}$$

$$N_6 = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 & 0 & -6 \\ 0 & 4 & 0 & 5 & 0 & -4 & 0 \\ 0 & 0 & 2 & 3 & -2 & 0 & 0 \\ 0 & 5 & 3 & 1 & 3 & 5 & 0 \\ 0 & 0 & -2 & 3 & 2 & 0 & 0 \\ 0 & -4 & 0 & 5 & 0 & 4 & 0 \\ -6 & 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix}$$

$$N_7 = \begin{bmatrix} 6 & 0 & 0 & 7 & 0 & 0 & -6 \\ 0 & 4 & 0 & 5 & 0 & -4 & 0 \\ 0 & 0 & 2 & 3 & -2 & 0 & 0 \\ 7 & 5 & 3 & 1 & 3 & 5 & 7 \\ 0 & 0 & -2 & 3 & 2 & 0 & 0 \\ 0 & -4 & 0 & 5 & 0 & 4 & 0 \\ -6 & 0 & 0 & 7 & 0 & 0 & 6 \end{bmatrix}$$

Submission

- Individual assignment.
- Prepare 3 files: `nMatrix.m`, `nMatrixRec.m`, and `nMatrixMV.m`.
- Write your name in each file.
- Archive them: `<your name>.zip` or `<your name>.tgz`
- Submit the archive by email to `pauldj@aices.rwth-aachen.de`
- Email's subject: ``LSC-18 Challenge2 <your last name>''
- **Deadline: Thursday, May 24, 11:59pm.**