Parallel Programming

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Collective Communication: Lower Bounds

Cost of communication: $\alpha + n\beta$

Cost of computation: $\gamma \, \#\mathrm{ops}$

$\alpha =$ "latency", "startup"

$n =$ size of the message

$p =$ # of processes

$\beta =$ 1/"bandwidth"

$\gamma =$ cost of 1 flop

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Primitive	Latency	Bandwidth	Computation
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`Reduce`	$\lceil \log_2(p) \rceil \alpha$	$n\beta$	$\frac{p-1}{p} n\gamma$

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`Scatter`	$\lceil \log_2(p) \rceil \alpha$	$\frac{p-1}{p} n\beta$	-
`Gather`	$\lceil \log_2(p) \rceil \alpha$	$\frac{p-1}{p} n\beta$	-

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`Gather`	$\lceil \log_2(p) \rceil \alpha$	$\frac{p-1}{p} n\beta$	-
`Allgather`	$\lceil \log_2(p) \rceil \alpha$	$\frac{p-1}{p} n\beta$	-
`Reduce-Scatter`	$\lceil \log_2(p) \rceil \alpha$	$\frac{p-1}{p} n\beta$	$\frac{p-1}{p} n\gamma$

Implementation of Bcast and Reduce

- IDEA: recursive doubling / "Minimum Spanning Tree" (MST)
  At each step, double the number of active processes.

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  - # steps: $\log_2 p$
  - cost(step): $\alpha + n\beta$
  - total time: $\log_2(p)\alpha + \log_2(p)n\beta$ lower bound: $\log_2(p)\alpha + n\beta$
  - note: cost(p^2) = 2 cost(p)!

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- Reduce
  BCast in reverse; cost(computation) ?

Implementation of Scatter (and Gather)

- IDEA: MST again
  At step i, only $\frac{1}{2^i}$-th of the message is sent
- # steps: $\log_2 p$
- cost(step$_i$): $\alpha + \frac{n}{2^i}\beta$
- total time: $$\sum_{i=1}^{\log_2(p)} \alpha + \frac{n}{2^i}\beta = \log_2(p)\alpha + \frac{p-1}{p}n\beta$$
- lower bound: $\log_2(p)\alpha + \frac{p-1}{p}n\beta$ optimal!

A different implementation of Bcast

- IDEA: Scatter + cyclic algorithm (e.g., pass to the right)
- Cost?

Implementation of Allgather (and Reduce-scatter)

- IDEA: "Recursive-doubling" (bidirectional exchange)
  Recursive allgather of half data + exchange data between disjoint nodes.

Node$_0$	Node$_1$	Node$_2$	Node$_3$
v[0]			
	v[1]		
		v[2]	
			v[3]

$\Downarrow$

Node$_0$	Node$_1$	Node$_2$	Node$_3$
v[0]	v[0]		
v[1]	v[1]		
		v[2]	v[2]
		v[3]	v[3]

$\Downarrow$

Node$_0$	Node$_1$	Node$_2$	Node$_3$
v[0]	v[0]	v[0]	v[0]
v[1]	v[1]	v[1]	v[1]
v[2]	v[2]	v[2]	v[2]
v[3]	v[3]	v[3]	v[3]

- # steps: $\log_2 p$
- total time:

$$\sum_{i=1}^{\log_2(p)} \alpha + \frac{n}{2^i}\beta = \log_2(p)\alpha + \frac{p-1}{p}n\beta$$

Another implementation of Allgather

- IDEA: Cyclic algorithm

Node$_0$	Node$_1$	Node$_2$	Node$_3$
v[0]			
	v[1]		
		v[2]	
			v[3]

$\Downarrow$

Node$_0$	Node$_1$	Node$_2$	Node$_3$
v[0]	v[0]		
	v[1]	v[1]	
		v[2]	v[2]
v[3]			v[3]

$\Downarrow$

Node$_0$	Node$_1$	Node$_2$	Node$_3$
v[0]	v[0]	v[0]	
	v[1]	v[1]	v[1]
v[2]		v[2]	v[2]
v[3]	v[3]		v[3]

- # steps: $p - 1$
- total time:

$$\sum_{i=1}^{p-1} \alpha + \frac{n}{p}\beta = (p-1)\alpha + \frac{p-1}{p} n\beta$$