Armadillo: Template Metaprogramming for Linear Algebra

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Seminar on Automation, Compilers, and Code-Generation
HPAC

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- ▷ C++ Library for Linear Algebra
- ▷ Aims for ease of use
- ▷ Still wants performance
- ▷ Abstract interface to BLAS and LAPACK
- ▷ Pure template library (only headers)

Code example

Calculating $A^{-1}Bvv^T$:

```
mat     A = << 3.0 << 2.5 << endr
            << 2.0 << 1.0 << endr
mat     B = randu(2,5);
colvec  v = randu(5);

mat result = inv(A)*B*v*trans(v);

cout << "Result:" << endl << result.row(0) << endl;
```

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- Therefore: no dynamic typing
- Allows for further optimizations by compiler

C++ Template example

Type-dependent:

```
class Adder{
    int base;
    public:
    Adder(int x):base(x){}

    int doit(int y){
        return base+y;
    }
};
```

Type-agnostic:

```
template<typename TYPE>
class Adder{
    TYPE base;
    public:
    Adder(TYPE x):base(x){}

    TYPE doit(TYPE y){
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```

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- ▷ Template used inside template → Recursion
- ▷ Branching and Recursion → **Turing complete**
- ▷ Allows offloading of computation to compile-time

Factorial: normal C++

```
int factorial(int n) {
    if(n < 1){
        return  1;
    }
    else {
        return n*factorial(n-1);
    }
}
```

Factorial: Template metaprogramming

```
template <int n>
struct factorial {
    const static int result =
                    n * factorial<n - 1>::result;
};

template <>
struct factorial<0> {
    const static int result = 1 ;
};
```

- ▷ Types used for pattern matching.
- ▷ Structs used as functions.
- ▷ Most "specific" template chosen first. (cf. Haskell)

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  e.g.: `diagmat(A)` or `trimatu(A)`
- Actual calculation delayed until runtime request for results

Simple example: `inv(inv(A))`

- Rationale: should not compute anything; just return `A`
- Inner call to `inv()` returns a `Op<A, op_inv>` object.
- Outter call matches the definition of `inv()` with `Op<A, op_inv>` argument. Returns `A`.

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- Outter call matches the definition of `inv()` with `Op<A, op_inv>` argument. Returns `A`.
- Advantages:
  - Obvious: no actual inversion.
  - Not obvious: multiple-dispatch without the runtime overhead.
  - Allows compiler to get rid of unneeded function calls.

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- $A \in \mathbb{R}^{\mathbf{60} \times \mathbf{5}}$, $B \in \mathbb{R}^{\mathbf{5} \times \mathbf{30}}$, $C \in \mathbb{R}^{\mathbf{30} \times \mathbf{10}}$
  - $(AB)C = (60 \cdot 5 \cdot 30) + (60 \cdot 30 \cdot 10) = 27.000$
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- E.g. Matlab: naive left to right ordering
- Complexity: $\mathcal{O}(n^3)$ with dynamic programming
  Smarter approach: $\mathcal{O}(n \cdot log(n))$

Complex example: A*B*C*D

- Instantiated as:
  `Glue<Glue<Glue<A,B,op_times>,C,op_times>,D,op_times>`
- Determines "depth" of expression: $N = 3$
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- Remember: all this at **compile-time**!

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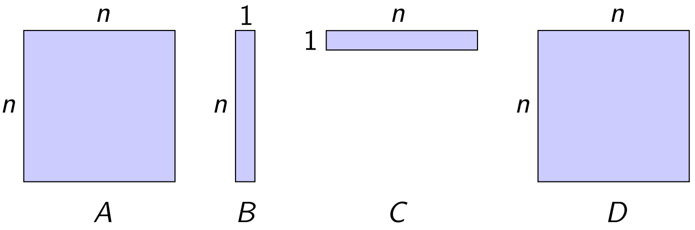

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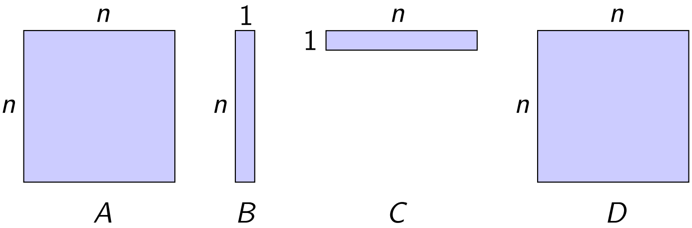

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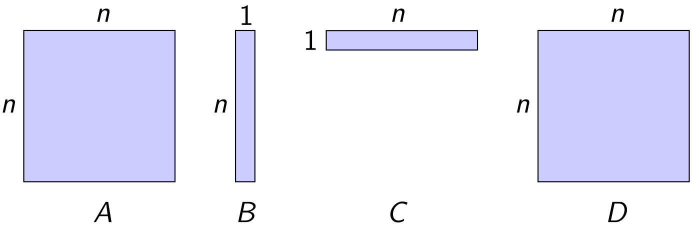
n	Armadillo	All
3	2	2
4	4	5
5	8	14
6	16	42
7	32	132



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- Lots more!

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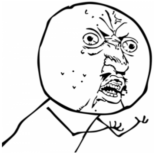

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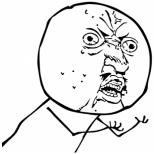

- ▷ Our recommendation: Use it! Don't try to understand it.

Thanks! Questions?

Sources:

- Conrad Sanderson, *Armadillo: An Open Source C++ Linear Algebra Library for Fast Prototyping and Computationally Intensive Experiments*, Technical Report, NICTA, 2010.
- Armadillo source code.
- *C++ Programming*, chapter *Template Meta-Programming*, Wikibooks
- *Matrix chain multiplication*, Wikipedia