Data Dependence and Vectorization

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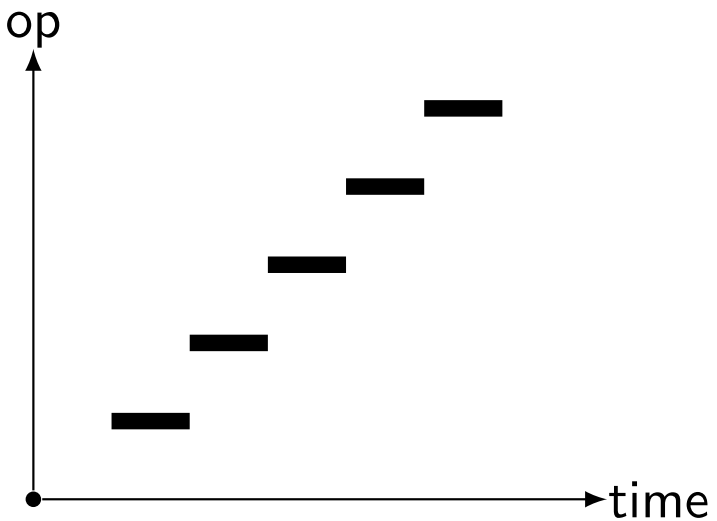
July 23, 2014

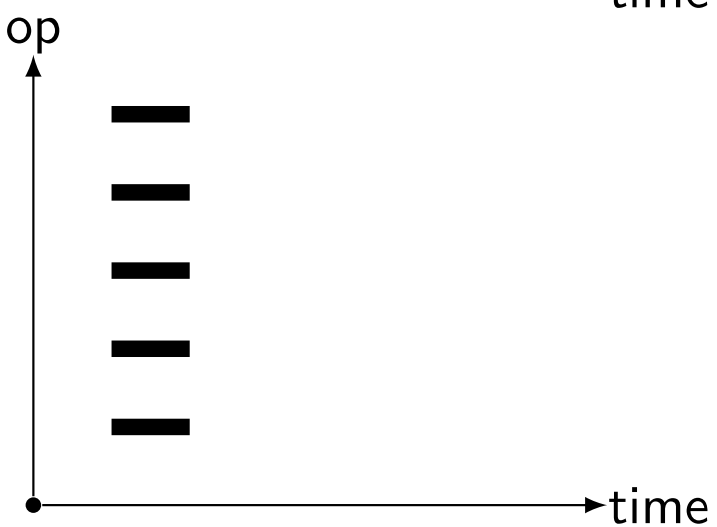
Contents

Vectorization - Basic Idea

```
DIMENSION Arr(5)
DO I=1,5
  Arr(I) = 0
END DO I
```




```
DIMENSION Arr(5)
Arr(1:5) = 0
```



Problem

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DIMENSION Arr(5)
DO I=1,4
  Arr(I+1) = Arr(I)+1
END DO I
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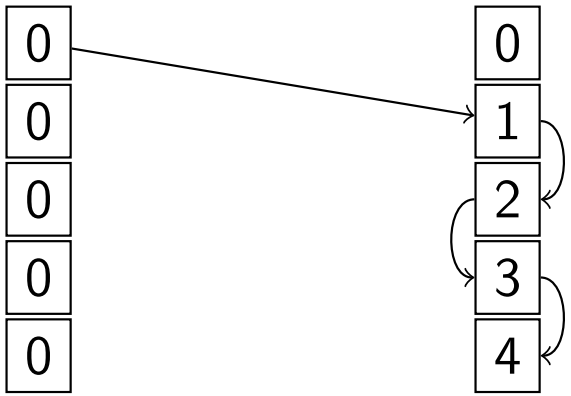

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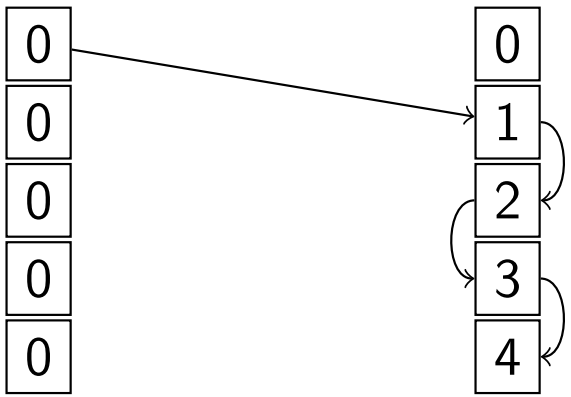
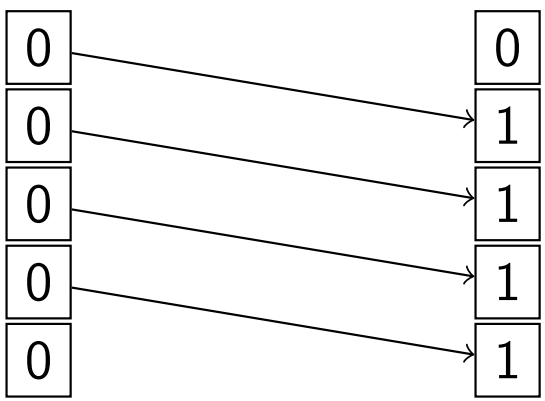



```
DIMENSION Arr(5)
Arr(2:5) = Arr(1:4)+1
```



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Program Model

Variables

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INTEGER I, J
DIMENSION Arr(100),
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Statements

S_1 `I=I+1`
S_2 `Arr(I)=0`
S_3 `Brr(1,1:100)=Arr(1:100)`

Program Model

Variables

```
INTEGER I, J
DIMENSION Arr(100),
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```

Do-Loops

```
DO I=0,100
 DO J=10,1,-1
 ...
 END DO J
END DO I
```

Statements

S_1 I=I+1
S_2 Arr(I)=0
S_3 Brr(1,1:100)=Arr(1:100)

Contents

Dependence Types

True dependence:

$$S_1 \delta^t S_2$$

$$\begin{array}{ll} S_1 & \mathsf{A} = \ldots \\ & \vdots \\ S_2 & \ldots = \mathsf{A} \end{array}$$

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Output dependence:

$S_1 \delta^o S_2$

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- $S_2 \delta^t S_3$

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- $S_2 \delta^t S_3$
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$S\delta S'$ is *loop-carried* if S is executed in a different iteration than S'.

All loop-independent dependencies are forward.

Dependence Types (2)

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      DO I=1,100
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  - backward
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All backward dependencies are loop-carried.

Dependence Tests

How to test for dependence automatically?

Separability Test

```
      DO I=1,100
S1      Arr(3+2*I)=...
        :
        :
S2      ... =Arr(-1+2*I)
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Equate subscript terms:

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Equate subscript terms:

$$a_0 + a * x = b_0 + b * y, \quad x, y \in \{lb, ub\}$$

Integer solutions within $\{lb, ub\} \rightsquigarrow$ Dependence!

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$$3 + 2 * x = -1 + 2 * y$$

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```

$$\Rightarrow \qquad \begin{aligned} 3 + 2 * x &= -1 + 2 * y \\ y &= x + 2 \end{aligned}$$

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Separability Test: Overview

Advantages:

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- necessary and sufficient criteria for dependence

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Consequence: Only used on single index vars, for multiple use gcd test.

gcd Test

```
   DO I=1,10
    DO J=1,5
S1    Arr(3I+6J-8) = ...
            :
            :
S2    ... = Arr(3I-3J+33)
    END DO J
   END DO I
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Build Diophantine equation:

$$3x_I + 6x_J - 3y_I + 3y_J = 25$$

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$gcd(3, 6, -3, 3) | 25 \rightarrow$ Independent!

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$gcd(3, 6, -3, 3) | 25 \rightarrow$ Independent! $\qquad$ $gcd(1, 2, -1, 3) | 33 \rightarrow$ Dependence?

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- computationally efficient

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$\Rightarrow$ no useful result

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   DO  I=1,10
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$S_1 \delta S_2$: loop-independent

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```

Crr(I):	Brr(1)	Brr(2)	Brr(3)	...

Why?

$S_2 \delta S_1$: loop-carried

$S_1 \delta S_2$: loop-independent

All loop-independent dependencies can be directly vectorized!

Statement Re-Ordering

```
   DO I=1,10
S1   Arr(I)=Brr(I)
S2   Crr(I)=Arr(I+1)
   END DO I
```

Statement Re-Ordering

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Crr(I):	Arr(2)	Arr(3)	Arr(4)	...

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```
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```

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Crr(I):	Arr(2)	Arr(3)	Arr(4)	...

```
S1   Crr(1:10)=Arr(2:11)
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Why?

$S_2 \delta S_1$: loop-carried backward

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```

Crr(I):	Arr(2)	Arr(3)	Arr(4)	...

Why?

$S_2 \delta S_1$: loop-carried backward

$S_2 \delta S_1$: loop-carried forward

All loop-carried forward dependent statements can be vectorized!

Vector Statement Generation (1)

```
   DO I=2,100
S1   Arr(I)=Drr(I-1)+Brr(I-1)
S2   Brr(I)=Arr(I-1)*2
S3   Crr(I)=Crr(I-1)+3
S4   Drr(I)=Crr(I)
   END DO I
```

Vector Statement Generation (1)

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   DO I=2,100
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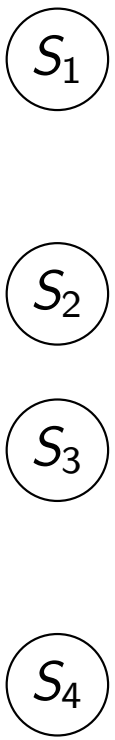
S4   Drr(I)=Crr(I)
   END DO I
```



Vector Statement Generation (1)

```
   DO  I=2,100
S1   Arr(I)=Drr(I-1)+Brr(I-1)
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   END DO  I
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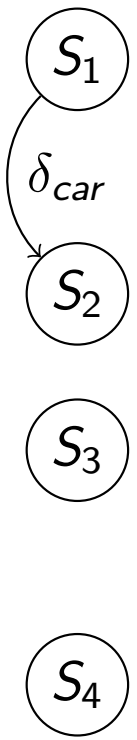

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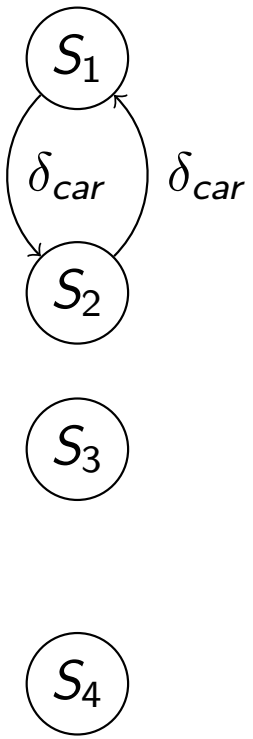

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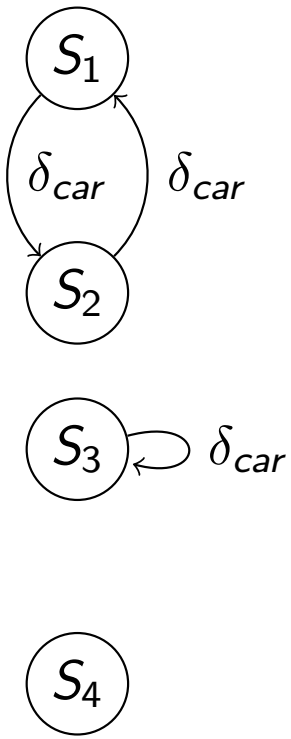

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   END DO I
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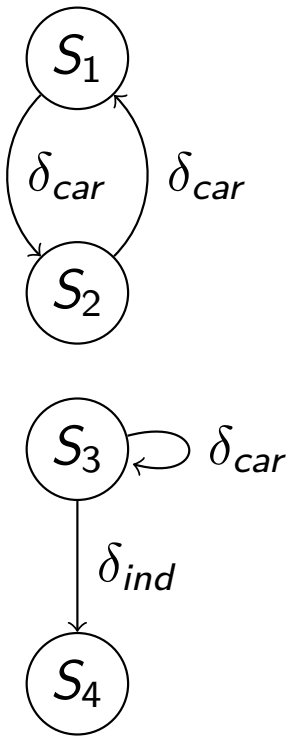

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   END DO I
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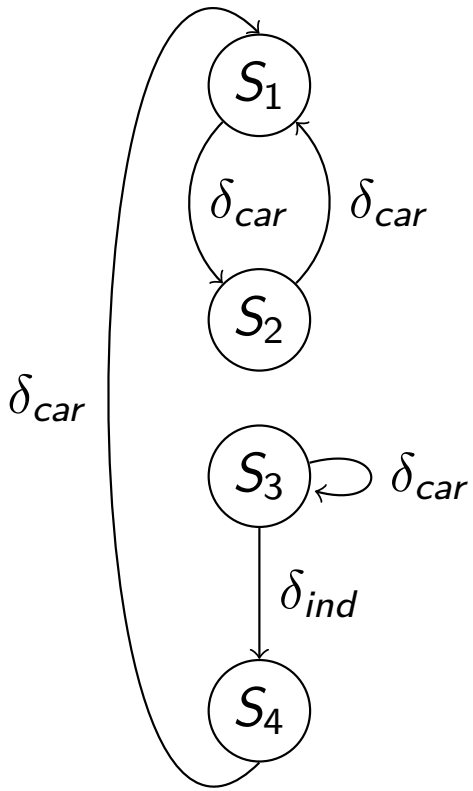

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   END DO I
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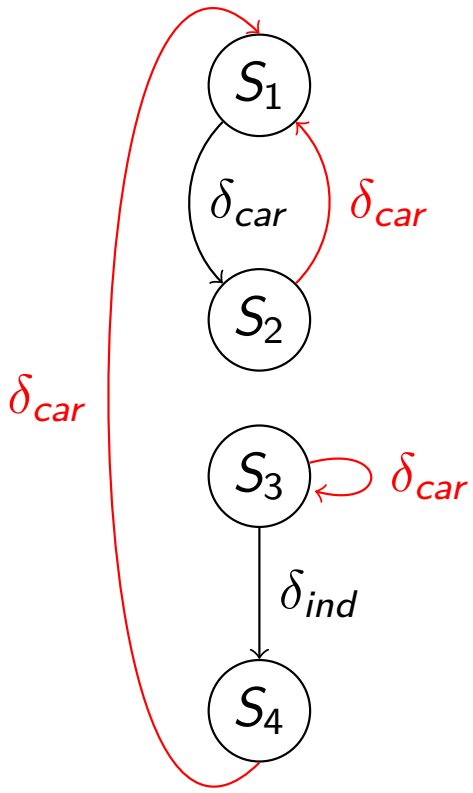

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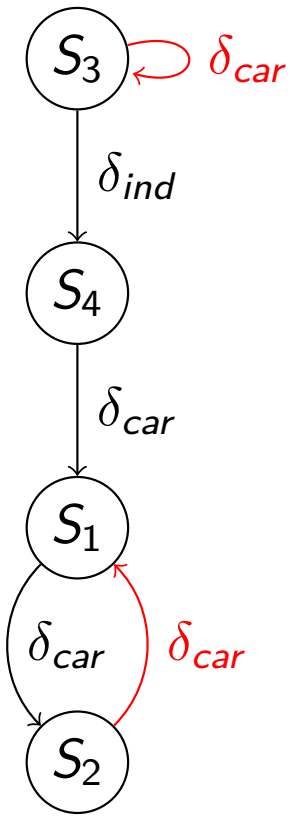

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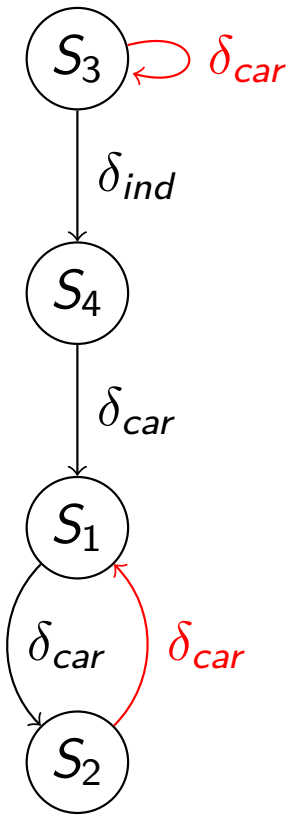
   END DO I
```

```
   DO I=2,100
S3   Crr(I)=Crr(I-1)+3
S4   Drr(I)=Crr(I)
S1   Arr(I)=Drr(I-1)+Brr(I-1)
S2   Brr(I)=Arr(I-1)*2
   END DO I
```

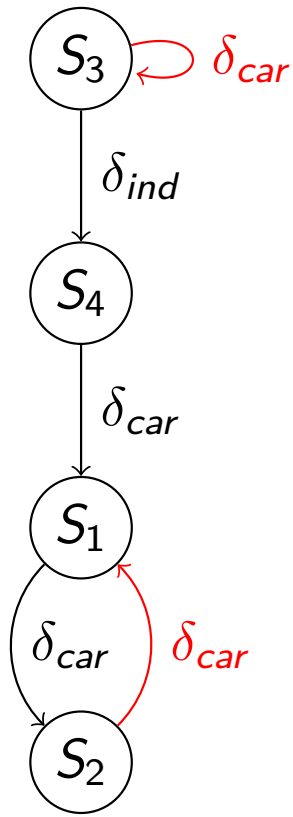


Vector Statement Generation (2)

```
   DO I=2,100
S3   Crr(I)=Crr(I-1)+3
   END DO I
```

```
S4 Drr(2:100)=Crr(2:100)
```

```
   DO I=2,100
S1   Arr(I)=Drr(I-1)+Brr(I-1)
S2   Brr(I)=Arr(I-1)*2
   END DO I
```



Partial Vectorization

```
   DO I=0,100
    DO J=0,100
S1    Arr(I,J) = 3 + Arr(I-1,J)
    END DO J
   END DO I
```

Partial Vectorization

```
   DO I=0,100
    DO J=0,100
S1    Arr(I,J) = 3 + Arr(I-1,J)
    END DO J
   END DO I
```

```
   DO I=0,100
S1  Arr(I,0:100) = 3 + Arr(I-1,0:100)
   END DO I
```

Contents

Do-Loop Index Normalization

```
DO I=10,0,-2
  Arr(I) = ...
END DO I
```

Do-Loop Index Normalization

Normalization steps:

1. introduce new variables

```
DO  I=10,0,-2
  Arr(I) = ...
END DO  I
```

```
DO  Î=1,(0-10+-2)/-2      |= 6
  Arr(10+(Î-1)*-2) = ...
END DO  Î
Î=10+MAX(6,0)*-2          |= -2
```

Do-Loop Index Normalization

Normalization steps:

1. introduce new variables
2. replace all occurrences of original index variables

```
DO  I=10,0,-2
  Arr(I) = ...
END DO  I
```

```
DO  Î=1,(0-10+-2)/-2      |= 6
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```

Do-Loop Index Normalization

Normalization steps:

1. introduce new variables
2. replace all occurrences of original index variables
3. iterate from 1 to N in steps of 1

```
DO I=10,0,-2
  Arr(I) = ...
END DO I
```

```
DO Î=1,(0-10+-2)/-2     |=6
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END DO Î
Î=10+MAX(6,0)*-2         |=-2
```

Do-Loop Index Normalization

Normalization steps:

1. introduce new variables
2. replace all occurrences of original index variables
3. iterate from 1 to N in steps of 1
4. ensure that postcondition is fulfilled

```
DO  I=10,0,-2
  Arr(I) = ...
END DO  I
```

```
DO  Î=1,(0-10+-2)/-2      |= 6
  Arr(10+(Î-1)*-2) = ...
END DO  Î
Î=10+MAX(6,0)*-2          |= -2
```

Scalar Forward Substitution

```
DO I=1,10
  g=4+3*I
  Arr(g) = ...
END DO I
```

Scalar Forward Substitution

Situation:

- non-index var g is used as subscript

```
DO  I=1,10
  g=4+3*I
  Arr(g) = ...
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```

Scalar Forward Substitution

Situation:

- non-index var g is used as subscript
- g depends only on index var

```
DO  I=1,10
  g=4+3*I
  Arr(g) = ...
END DO  I
```

Scalar Forward Substitution

Situation:

- non-index var g is used as subscript
- g depends only on index var

Solution:

- replace uses of g by its definition

```
DO I=1,10
  g=4+3*I
  Arr(g) = ...
END DO I
```

```
DO I=1,10
  g=4+3*I
  Arr(4+3*I) = ...
END DO I
```

Induction Variable Substitution

```
INTEGER g=5
DO I=1,10
  g=g+3
  Arr(g+4) = ...
END DO I
```

Induction Variable Substitution

Situation:

- non-index var g is used as induction var

```
INTEGER g=5
DO I=1,10
  g=g+3
  Arr(g+4) = ...
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Situation:

- non-index var g is used as induction var
- g's definition is of the form: $g = g + c$

```
INTEGER g=5
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  g=g+3
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END DO I
```

Induction Variable Substitution

Situation:

- non-index var g is used as induction var
- g's definition is of the form: $g = g + c$

Solution:

- replace uses of g by: $g_{base} + c * I$

```
INTEGER g=5
DO I=1,10
  g=g+3
  Arr(g+4) = ...
END DO I
```

```
INTEGER g=5
DO I=1,10
  g=g+3
  Arr(5+3*I+4) = ...
END DO I
```

References


D. J. Kuck, R. H. Kuhn, D. A. Padua, B. Leasure, and M. Wolfe.
Dependence graphs and compiler optimizations.
In *Proceedings of the 8th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages*, POPL '81, pages 207–218, New York, NY, USA, 1981. ACM.


H. Zima and B. Chapman.
Supercompilers for Parallel and Vector Computers.
ACM, New York, NY, USA, 1991.