

Introduction to GAP

(Groups, Algorithms, Programming)

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Seminar Language in Scientific Computing
24.01.2013

Outline

- What is GAP?
- Symbolic computation
- Group theory
- History
- The GAP System
- Demo & Functionality
- Bibliography
- Q & A

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What is GAP?

GAP is:

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GAP is:

- a system for computational discrete algebra (Computational Group Theory)—that provides:

What is GAP?

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- a system for computational discrete algebra (Computational Group Theory)—that provides:
 - a language;

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 - a language;
 - a library;

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 - a language;
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What is GAP?

GAP is:

- a system for computational discrete algebra (Computational Group Theory)—that provides:
 - a language;
 - a library;
 - a data library
- A symbolic computation/ computer algebra/ algebraic computation *tool*

What is GAP?

Capabilities:

- **Forming** lists, records, domains
- **Handling** objects and elements, Booleans, orderings, mappings, and relations
- **Computing** integers, rational numbers, cyclotomic numbers(Gaussian numbers), finite fields, p -adic numbers, polynomials, rational functions, group elements, permutation, matrices, abstract words
- etc

Symbolic Mathematics/ Computing vs Numerical Mathematics/ Computing

- Symbolic Computing – Definition:

Wikipedia: *scientific area that refers to the study and development of algorithms and software for manipulating mathematical expressions and other mathematical objects*

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- What can be considered as of symbolic computing:
 - symbolic mathematical operation (integration, summation, solution of differential equation, ..)
 - Polynomial factorisation
 - etc

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 - etc
- Good for teaching mathematics!

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Group theory

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- Studies *algebraic structures* known as **group**

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- Applications in:
 - Chemistry;
 - Physics;
 - Cryptography, etc

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History of GAP (1)



- 1986: developed first by Lehrstuhl D für Mathematik, RWTH Aachen: Joachim Neubüser, Johannes Meier, Alice Niemayer, Werner Nickel, Martin Schönert
- 1988: first publicly released version: GAP 2.4
- 1997: development in CIRCA, St Andrews, Scotland , UK 
- 2008: awarded the ACM/SIGSAM Richard Dimick Jenks Memorial Prize for Excellence in Software Engineering applied to Computer Algebra



History of GAP (2)

- 2012: Latest version GAP 4.5.7
- Originally supported by DFG (since 1992)
- Developed now by a group of researchers in:



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The GAP systems

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- Packages: ca 100 packages, act as algorithms or methods or libraries

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- Data Library:

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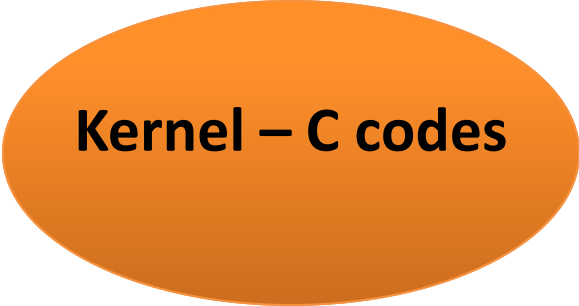
- **The kernel is written in C!**

GAP codes

libraries

GMP

Kernel – C codes



Kernel – C codes

Implemented in GAP
language



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Interactive environment in
developing and using GAP



Kernel – C codes

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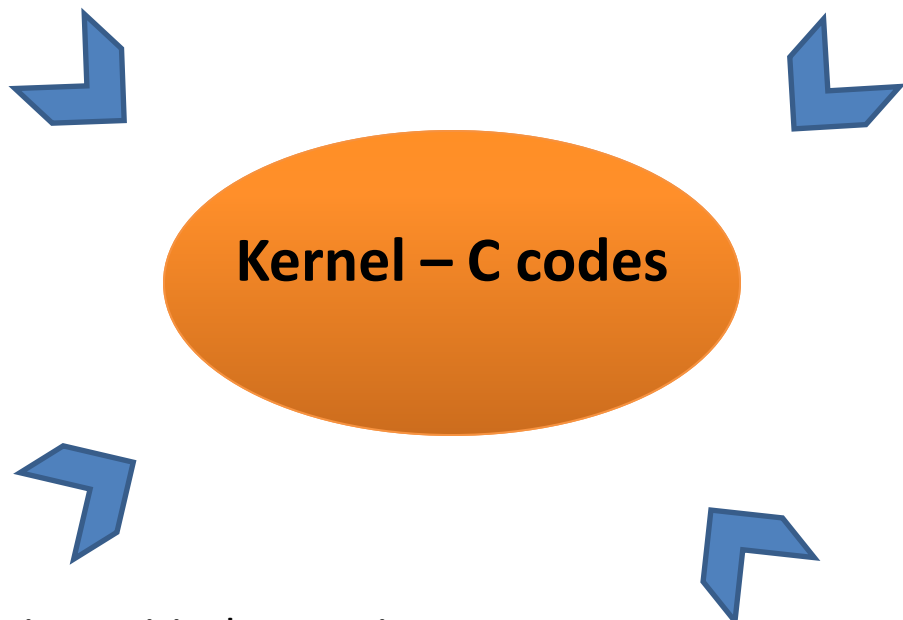
Kernel – C codes



Memory
management

Implemented in GAP
language

Interactive environment in
developing and using GAP



Kernel – C codes

fast versions of time critical operations
for various data types

Memory
management

GAP codes

GAP codes

- Pascal dialect
- using streams, lists, and records
- automatic memory management, garbage collection!

GAP codes

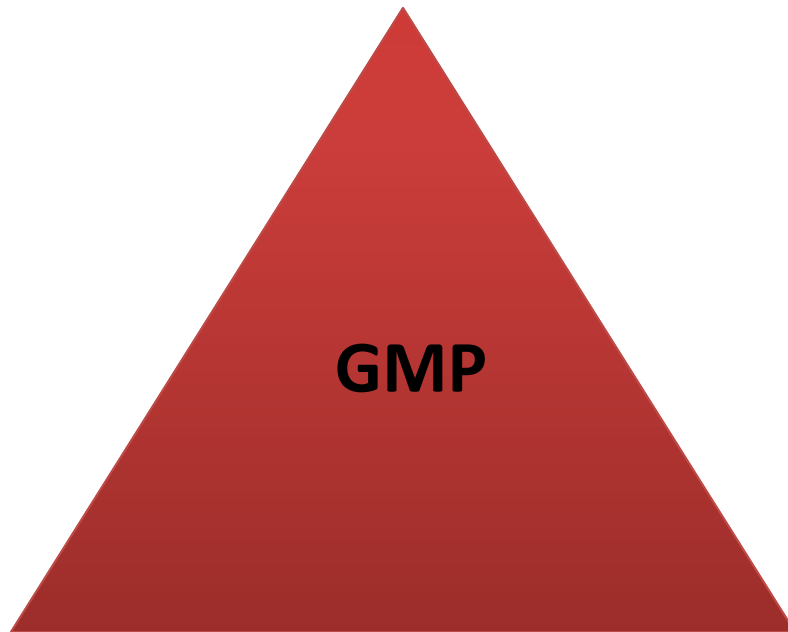
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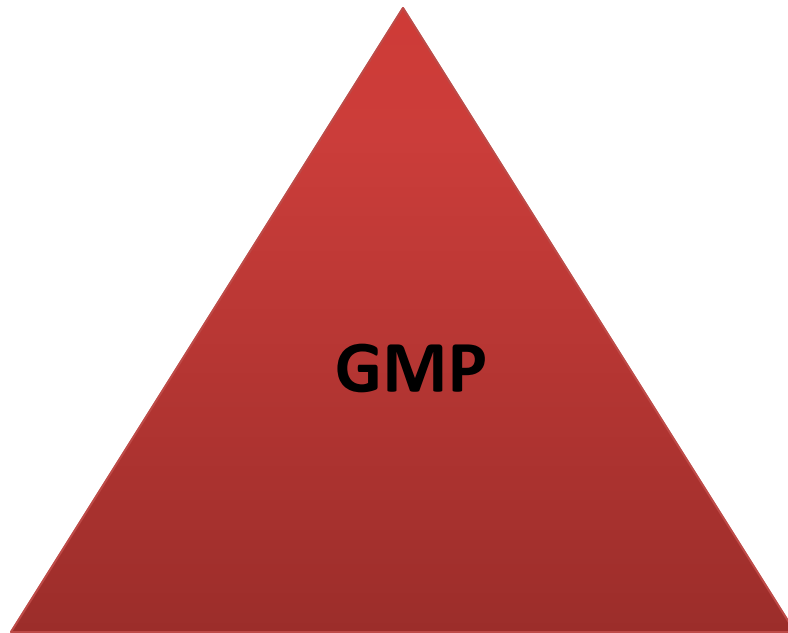
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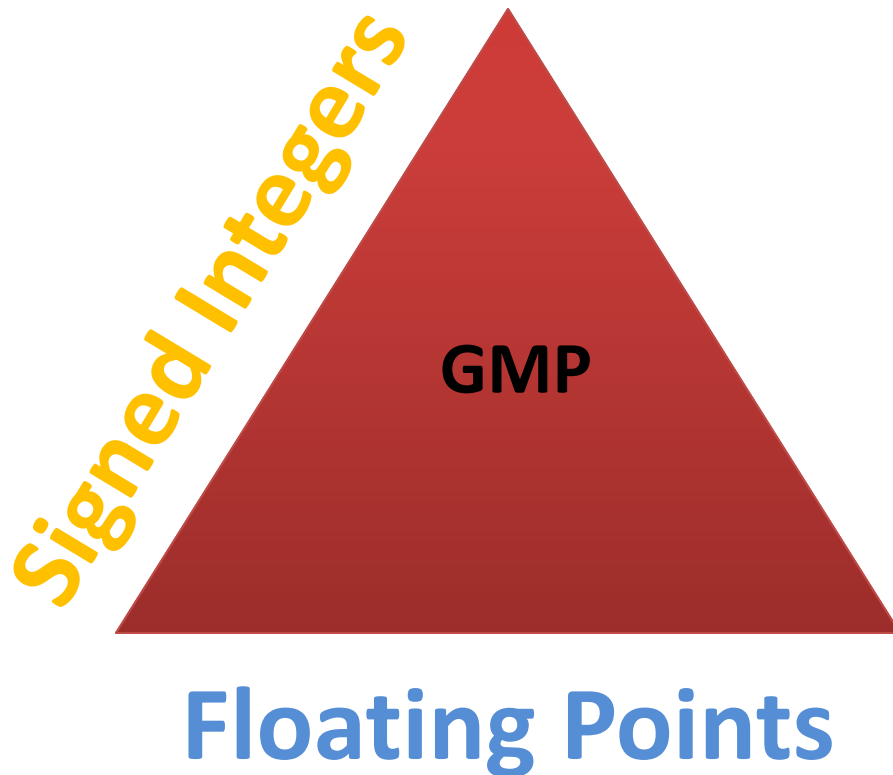


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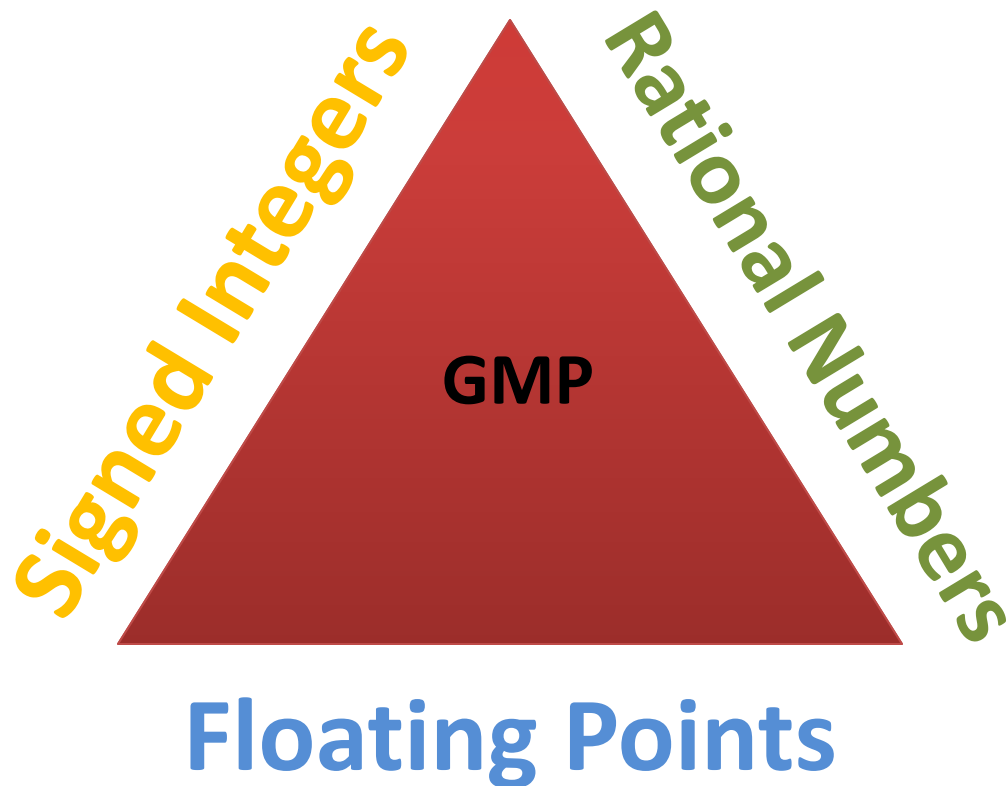


Floating Points

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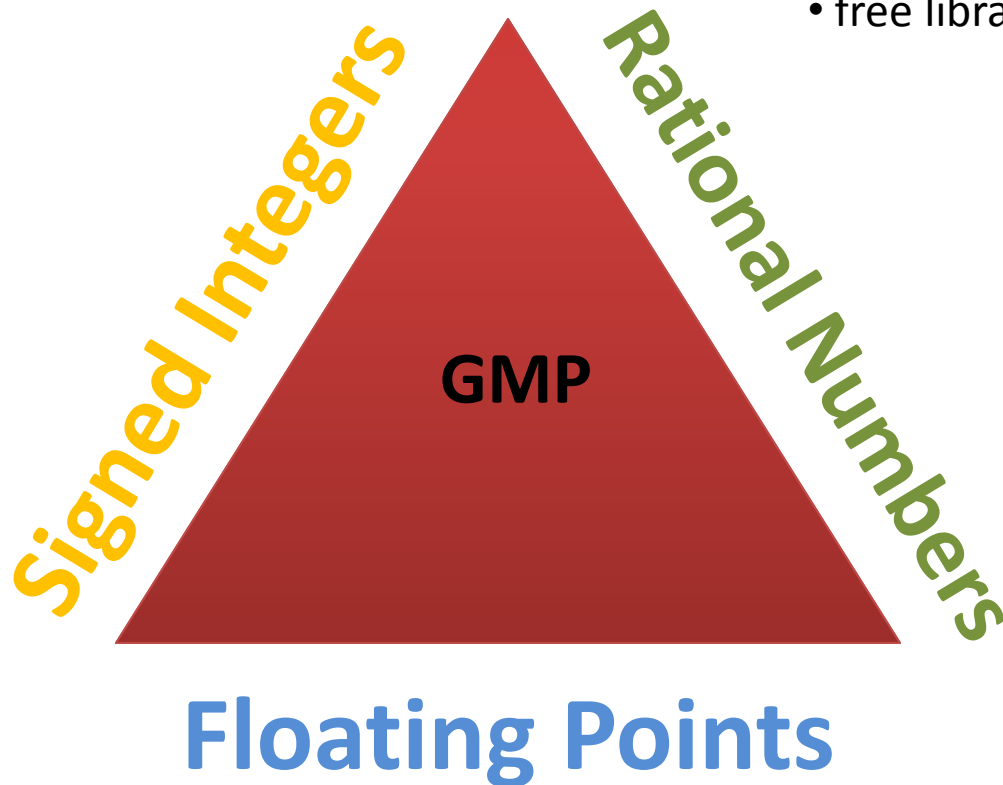


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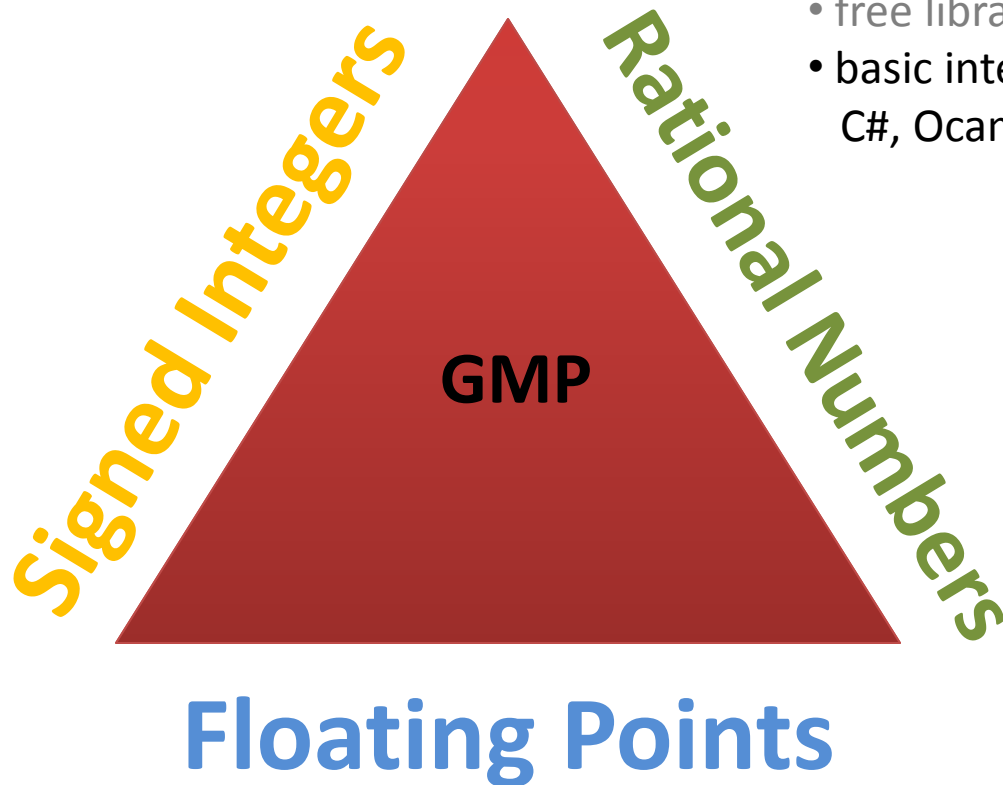
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- free library



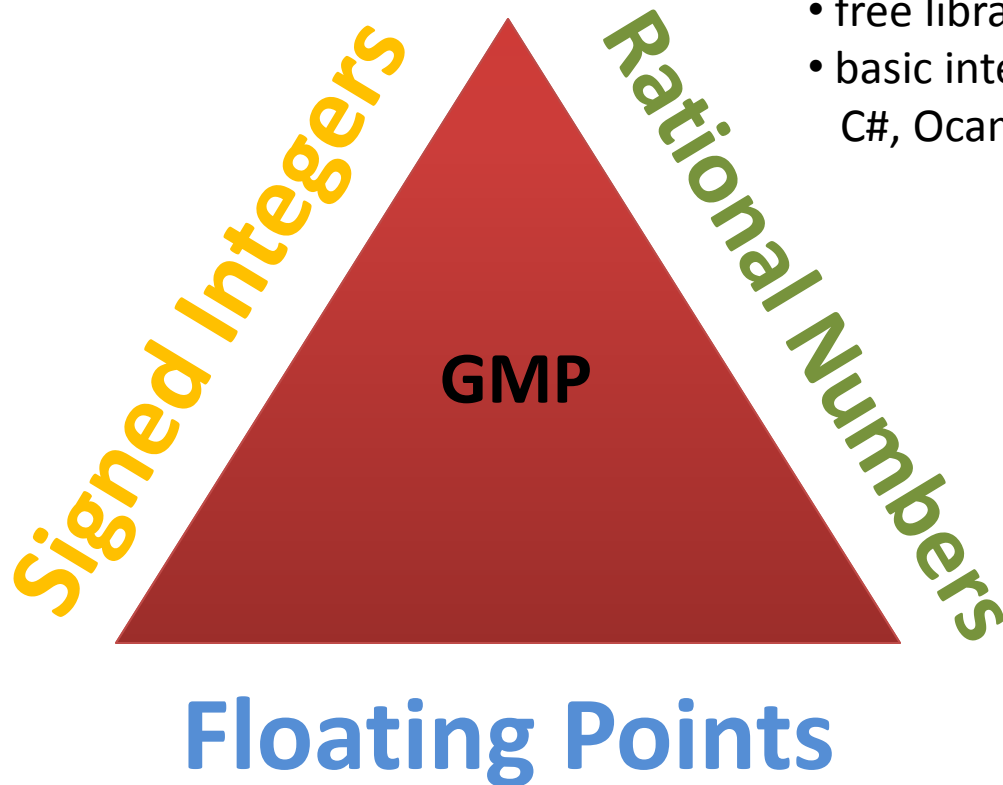
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libraries

- Support mathematical operations: basic algebra, group theory, etc

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for providing further information on GAP