Chaotic Systems as Compositional Algorithms

A presentation on Algorithmic Composition

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RWTH Aachen

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Presentation outline

Section 1

What is Chaos?

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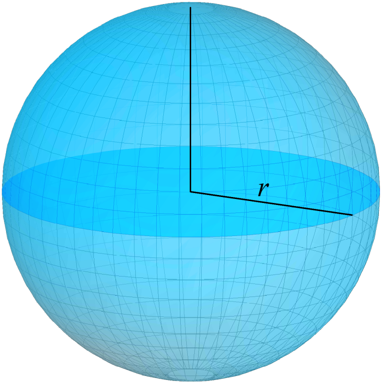
Natural shapes seem complex...

What is Chaos?



While typical mathematical shapes are simple and boring.

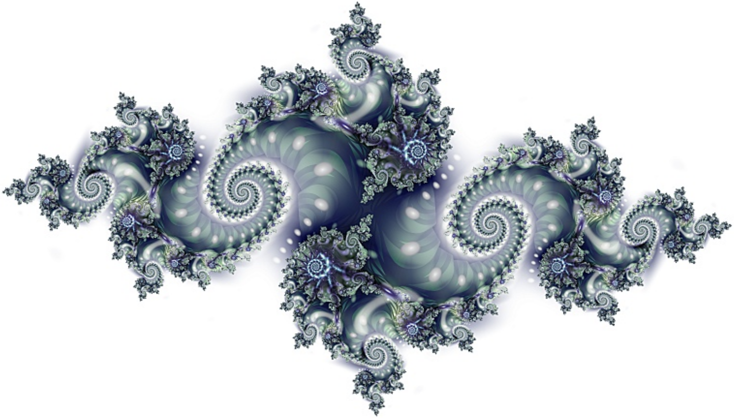
What is Chaos?

"Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in straight lines."
(Benoit Mandelbrot)

What is Chaos?

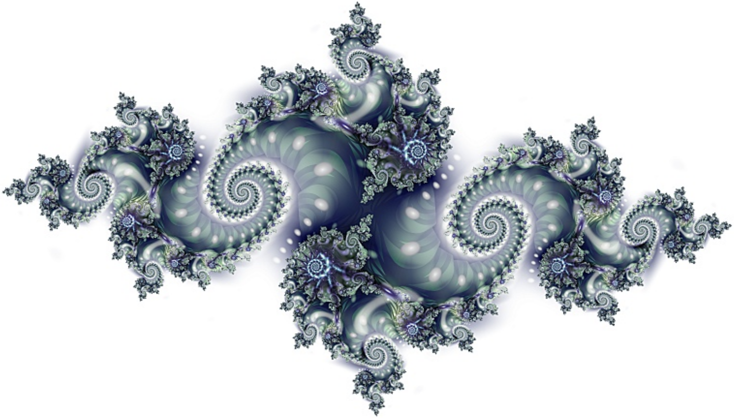
Chaos is about the realisation that even simple systems can exhibit complex behaviour.

What is Chaos?



It provides us with a link between visible complexity and mathematical simplicity.

What is Chaos?



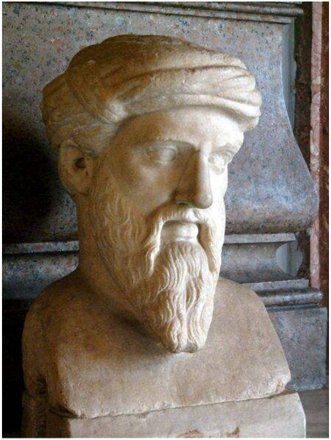
$f_c(z) = z^2 + c$, where $z, c \in \mathbb{C}$

Section 2

Nature and Music

Musica Universalis

- To ancient philosophers music played an essential role in the universe.



Pythagoras

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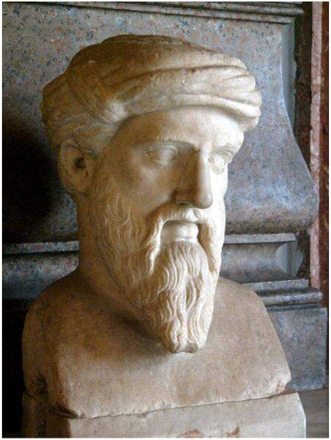

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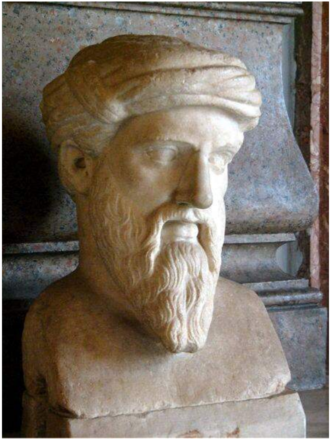
- “As in heaven, so on Earth.”



Pythagoras

The Scales and the Planets

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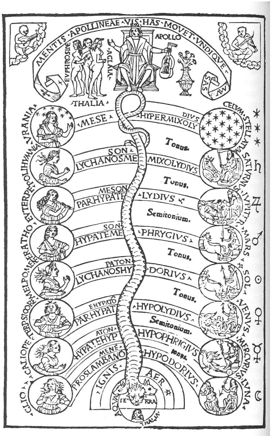

Medieval illustration of “The Music of the Spheres”

The Scales and the Planets

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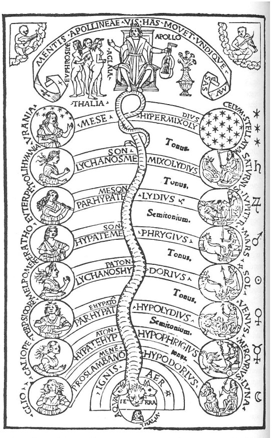

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The Scales and the Planets

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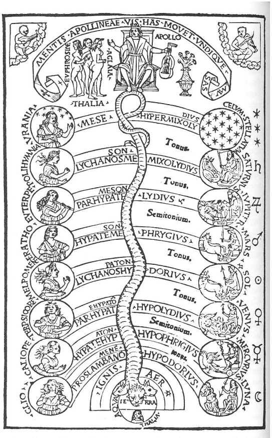
- The underlying idea remained:
- The relations in music were able to capture some essential property of nature.



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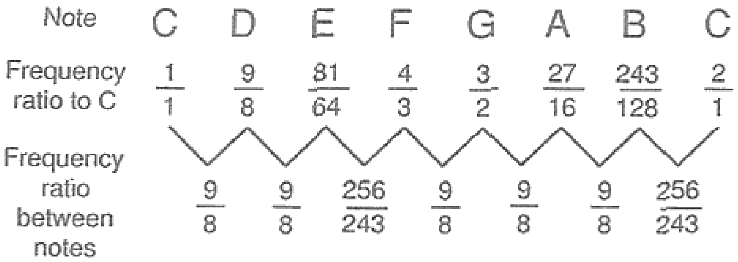
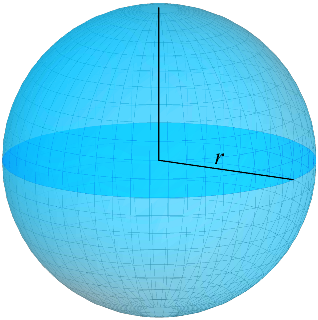


The Pythagorean Scale

Limitations

- The greeks and their successors were restricted by Euclidean conceptions of nature.



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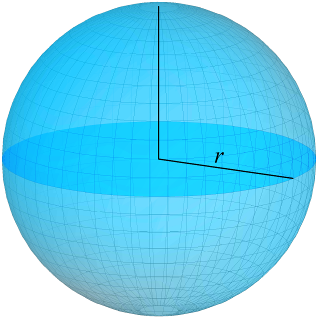

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- They admired regularity.
- As we have seen, however, this is not enough to capture the complexity of nature.

Section 3

Nature and Chaos

Dynamics around us

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- Given a state, the rules compute the next one.
- By induction, if we know one state we know all states.
- Therefore, dynamic systems are fully deterministic, given a starting state.

Dynamical systems

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- An orbit that eventually repeats is called periodic.
- One that does not is called aperiodic.

Chaos, formalized

- Sensitivity to initial conditions

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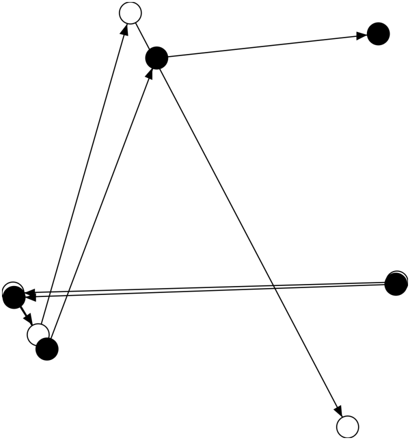
- Dense periodic orbits

Sensitivity to initial conditions



$$x_{n+1} = 4x_n(1-x),\ y_{n+1} = x_n + y_n \bmod 1$$

Topological Mixing

Dense Periodical Orbits

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- Infinitesimal perturbations throw system off
- "Butterfly effect"

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- Chaotic systems are fully deterministic;
- However, minor perturbations lead to huge differences.
- Often floating-point roundoff is enough.
- Effectively impossible to predict how the system will evolve.

Deterministic but not that much

"Chaos: When the present determines the future, but the approximate present does not approximately determine the future."
(Edward Lorenz)

Order within Chaos

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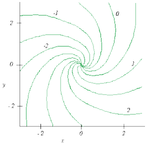

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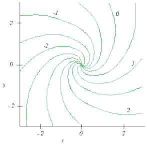

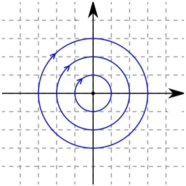

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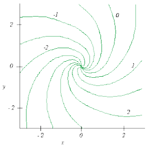
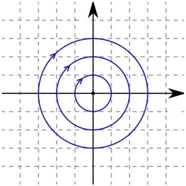
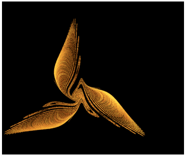
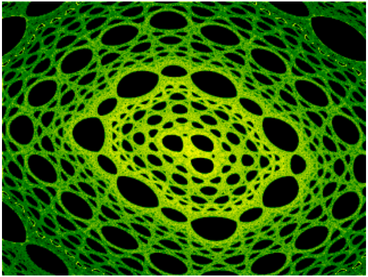
- Systems that model nature must include energy loss.
- Orbits contract onto an attractor.
- With no chaos, this will be a point (orbit of period 1)...
- Or a set of points (period > 1)...
- But with chaos...

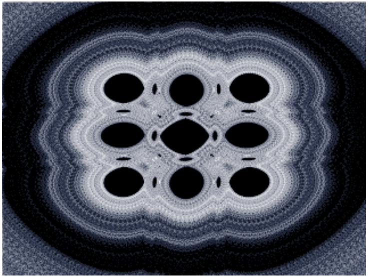


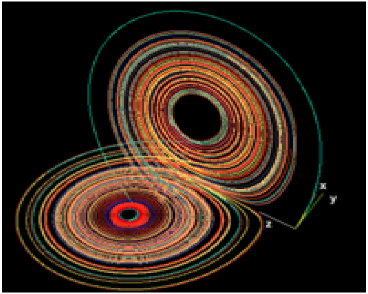


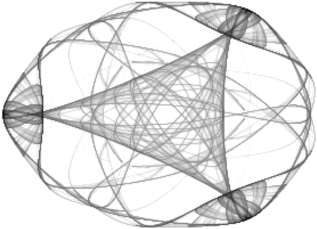


Strange attractors









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- Extracting music from chaos allows us to embed these aspects into music.

Section 4

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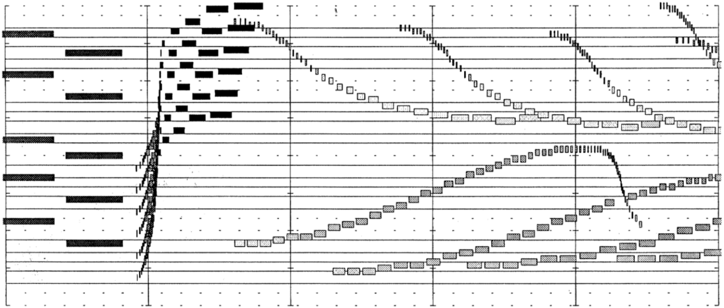
Mapping Chaos

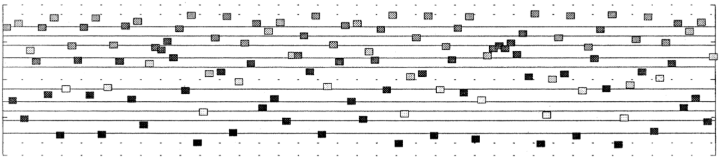


Excerpt from Rick Bidlack's "Dodecanon 1", created from a single orbit of the Lorenz Attractor

Mapping Chaos

- How to map the state vectors into notes depends on the artist's preference.
- Most common: One dimension of the vector $\rightarrow$ One musical parameter

Mapping Chaos



An orbit extracted from the Hènon Attractor using a very different mapping technique

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- Some composers have criticized this and proposed chaos for sound synthesis.
- Ex: Strange Attractor (iOS app)

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Conclusion

- Chaos is a powerful mechanism for the generation of musical phrases.
- It can also allow us to give music a “natural” quality.
- The properties of chaotic orbits allow for easy generation of infinite variation.
- However, large-scale use of chaos is still very composer-dependant.
- Small-scale use of chaos in sound synthesis is a promising field.

Sources

- Rick Bidlack. Chaotic Systems as Simple (but Complex) Compositional Algorithms, 1992
- John A. Maurer iv. The Influence of Chaos on Computer-Generated Music, 1999
- Benoit Mandelbrot. The Fractal Geometry of Nature, 1982
- Saber N. Elaydi. Discrete Chaos, 1999