# Chaotic Systems as Compositional Algorithms **MUS-15**

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## Introduction

The seemingly mystical ability of music to evoke feelings and images has fascinated man since times immemorial. The idea that the harmony of sounds as expressed in music might capture some essential property of the universe is at least as old as Greek philosophy, being expressed in the thoughts of Plato, Ptolemy, Pythagoras and others. [10] [5] In more recent times, the theory of Chaos has cast light on the mathematical principles underlying many processes in nature, such as the evolution of weather in the atmosphere and the diffusion of substances in fluids, and its mathematical cousin, the theory of Fractal Geometry, has illuminated us on how such seemingly random processes generate the complexity that surrounds us. [8]

Chaos, for the purposes of this report, describes the output, under certain conditions which will be discussed further on, of nonlinear dynamical systems. (Although it has been demonstrated that it is possible to obtain chaotic outputs even when dealing with linear systems, such possibilities are outside the scope of this report) The possibility of using such chaotic processes for the generation of music holds both artistic and philosophical interest: By the very nature of chaotic processes, they generate outputs which are, in a sense, "eternally new". This can be intuitively understood by watching the evolution of the ever-changing shape of a flame or a cloud, both generated by natural chaotic processes. Using the outputs of chaotic systems for automated composition allows for the generation of music which exhibits endless variation within the limits of the scales utilized. On the philosophical side, given that chaos is a fundamental cornerstone of many natural processes, using chaos to generate music allows us to embed qualities of nature within music, in a way realizing the ideas of ancient philosophers regarding the possibility of expressing the harmony of nature through music. It allows us to produce the musical equivalents of the aforementioned shapes of clouds or flames.

We will describe a method by which musical phrases can be extracted from dynamical systems, allowing us to use chaotic systems as music-composing algorithms. For this purpose, we will discuss the philosophical and historical background behind the idea of representing nature through music, followed by an informal discussion of the theory of dynamical systems and chaos and a brief presentation of the use of chaos theory in algorithmic music composition. We aim to show how the use of chaos in music composition can lead to musical phrases that have potentially endless variation, while forming complex patterns and motifs, and embodying certain "natural" qualities which could not be obtained through other methods of algorithmic composition.

## Music and Nature

A recurring concept along the history of mankind is the idea of "Musica Universalis" - or, translated, the "Music of the Universe". It was conjectured by many early philosophers that there was a harmony to the universe analogous to the harmony of music.[12] While such a concept would today be considered clearly unscientific, the idea that we might be able to endow music with properties of nature - or, in other terms, to produce the aural equivalent of a natural phenomenon, seems plausible: If we have a mathematical model of some natural process and a method by which this model might be used to generate musical events, it would become possible to generate music which is, in a manner defined by the method, or algorithm, by which the mathematical model's output is converted into music, an equivalent of the natural phenomenon in question.

To capture some fundamental property of nature through music is not a novel concept. The idea that the harmony of nature might be captured musically was explored extensively by Greek philosophers and revisited many times along the course of history. This was, for instance, a cornerstone of the Greek school of Pythagoreanism, whose belief that all that exists in the universe could be explained in terms of ratios of small integers led to the development of a musical scale where all intervals were based on the ratio 3/2, the so-called "Pythagorean tuning". [1]

However, attempts prior to the discovery of chaos theory were insufficient to truly capture natural phenomenons and shapes in music, as there was not a mathematical framework powerful enough to model the irregularity and complexity of most natural phenomena - specifically, until recent times, musicians were primarily concerned with regularity and order, in a way somewhat analogous to Euclidean geometry and to the Pythagoreans' preoccupation with ratios of integers. The Canadian composer Barry Truax says on this matter: "although we cannot say that the music of J.S. Bach is great because it is the aural equivalent of Cartesian geometry (...) we can hardly deny that it arises from the same *Zeigeist* or whatever one chooses to call the nexus of intellectual, cultural and aesthetic currents that influence an artist". [11]

It is not possible to express the forms and dynamics of nature through regularity, as these are, indeed, *defined* by irregularity - as said by Polish-born mathematician Benoit Mandelbrot, "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line". Mandelbrot himself described a theory of geometrical objects which were equally "rough" and complex at all scales, calling these objects "fractals" (from latin *fractus*, "broken"), due to their irregular nature.

Mandelbrot's fractal geometry is capable of reproducing the complexity found in natural forms, from the infinitely complex shape of a cloud to a leaf - and it is intimately tied to chaos, as shall be further explored in the next section.

## A Brief Definition of Chaos

As explorations of the applications of chaos to music will compose the remaining sections of this report, it is adequate here to give a slightly more formal definition of the term chaos. This will not be an in-depth explanation, as our objective here is simply to give the reader an intuitive understanding of chaos sufficient for them to comprehend the rest of this report. The reader is referred to an introductory-level textbook on dynamical systems in the case they wish to obtain a detailed understanding of chaotic systems.

### Dynamical systems

A dynamical system is a mathematical formalism describing the evolution of some system in time. A nonlinear dynamical system may be modeled as a system of equations operating on $\mathbb{R}^n$ for some $n \in \mathbb{N}^+$, which is iterated such that the solutions calculated from one iteration are fed back into the equations such that they become the input values for the next iteration. The current state after a particular iteration is then given by the output of that iteration. Notably, this formulation can be used to express both discrete-time systems or *maps* (notated using systems of difference equations) and continuous-time systems or *flows* (expressed with systems of differential equations).

A sequence of values generated in this iterative manner is called an *orbit*. A particular orbit of a dynamical system represents the behavior of the system for the particular set of initial conditions represented by the values which initiate it. As the set of initial conditions is equal to $\mathbb{R}^n$, the number of possible orbits of a particular dynamical system is infinite. [2] An orbit is called *periodic* if after some number $n$ of states it starts repeating previously-visited states, and *aperiodic* if it is not periodic.

Dynamical systems which model processes in which energy is dissipated to the surrouding environment also exhibit the important property of being *dissipative*. This means that the system's phase space - this is, the subset of $\mathbb{R}^n$ containing all possible states - shrinks with time, eventually leading to the formation of an *attractor* after a transient phase. An attractor is a set of states toward which the system tends to evolve regardless of the starting state. More formally, an attractor is a set of states such that:

- If the state of the system is within the attractor, all states that follow it are also within the attractor.
- There exists a neighborhood of the attractor such that, if a state is within this neighborhood, it will be within the attractor some number of iterations later.
- There is no subset of the attractor that has the two properties above.

Due to the iterative nature of dynamical systems, under certain conditions which will be explored in the next section, the attractor set of a given system might have a shape that is self-similar and therefore fractal.

## Chaotic systems

A chaotic dynamical system is a dynamical system that exhibits *chaos* in some region of its phase space. For the purposes of this report, we are interested only in dynamical systems that exhibit chaos on their attractor, as these systems will generate non-repeating orbits, allowing us to extract musical phrases with "infinite" variation, only limited by the musical scales we choose to use.

A system exhibits chaos if it satisfies the following conditions:

- Sensitivity to Initial Conditions

  Sensitivity to initial conditions means that every point within the chaotic region of the system's phase space is arbitrarily closely approximated by points with significantly different trajectories. More formally, this means that, for two points in phase space that are arbitrarily close, with initial separation $\delta_0$, their separation after an interval $t$ is as follows: $|\delta_t| \approx |e^{\lambda t}\delta_0|$

  The exponent $\lambda$ is called the *Lyapunov exponent* of the system. A positive Lyapunov exponent is an indicator of chaotic behavior.

- Topological Mixing

  Topological mixing means that the system evolves over time so that any part of the chaotic region of phase space will eventually overlap all of that chaotic region. In our case, any region of the attractor will eventually overlap the entire attractor. This meaning of "mixing" corresponds to the intuitive meaning of the word, and mixing dyes is an example of a chaotic system.

- Dense Periodic Orbits

  For a system to have dense periodic orbits means that every point in its phase space is approximated arbitrarily closely by periodic orbits. This is important because it means that, even if the system seems to be on a periodic orbit, it only takes an infinitesimally small perturbation to disturb it - which, combined with sensitivity to initial conditions means that we can never be sure we are on a periodic orbit, as the system might start evolving unpredictably at any moment.

The three properties above, combined, mean that it is essentially impossible to predict how a chaotic system might evolve - Even though a chaotic system is described by fully deterministic rules. In fact, the perturbations required to change drastically the behavior of a chaotic system are so small that chaos was originally discovered due to floating-point roundoff causing tremendous differences in the output of a weather model. [6]

### Attractor Types

Under non-chaotic conditions, an attractor might be a single point [cf. figure 1 on page 7] (corresponding to a periodic orbit of length 1), or a set of points on some closed loop [cf. figure 2 on page 7] (corresponding to a periodic orbit of length $> 1$). However, chaotic systems have attractors with complex fractal shapes, which resemble in many ways natural shapes. [cf. figure 3 on page 7] Despite the fact that we cannot predict the orbit starting at any specific point, it is the fact that the orbit will remain confined to this attractor and the fractal form of the attractor itself which confer the “natural” character we are seeking to the generated music.

These chaotic, fractal-shaped attractors are called *Strange attractors*, and it is through them that we find our aforementioned link between the theories of chaos and of fractal geometry. From the shapes of strange attractors it is visible that unpredictable systems might indeed lead to great order. Despite us being unable to predict the trajectory of any given point upon the attractor, knowing that the state remains confined to this complex, natural-seeming shape allows us to come to an intuitive understanding of how chaotic processes generate regular, however ever-changing patterns. It is now simple to understand how music generated by chaotic orbits diverges from that generated by non-chaotic orbits. [cf. figure 4 on page 7]

## Music and Chaos

### Mapping Chaos to Music

The general technique employed to extract music from the output of chaotic systems has remained roughly the same throughout the history of the use of chaos for compositions: The state vector $\vec{x}_n$ is mapped to some musical property according to a composer-defined mapping. Analogously, an orbit can be mapped to a musical phrase of arbitrary length Early works focused mainly on one-dimensional systems, with the mapping being generally pitch-focused - the state value would be mapped to some note on a composer-chosen scale. [9] [4]

Examples of music properties that might be mapped from outputs of the chaotic system are:

- Note pitch
- Note duration
- Instrument dynamics (attack time, sustain, release time...)
- Rhythm
- Instrumentation

The exact properties modulated depend only on the composer's choices. Likewise, the exact mapping used to transform the outputs of the chaotic system into musical properties depends only on the composer. When mapping the system to pitches (the most commonly explored technique), it is common for the composer to select a scale to work on and then define a mapping from the phase space of the system into that scale.

Bidlack [2] pioneered the technique of mapping multi-dimensional state vectors in order to chaotically influence more than one musical property - for instance, a two-dimensional map's first dimension might be mapped to pitch, while its second is mapped to note length.

### Brief History of Chaotic Composition

We can cite as a few important pioneers of composition using chaos Jeff Pressing, Michael Gogins, and Rick Bidlack.

Pressing's work was one of the earliest to employ chaos in music generation. He focused on explorations of orbits that were on the border between periodic and chaotic behavior, generating musical phrases that went back and forth between quasi-periodic and chaotic. [9]

Gogins, in turn, innovated by using so called "Iterated Function Systems", where instead of a single function being applied to the state of the system at each iteration, a function is randomly chosen from a set of several functions with predefined probabilities. This allows one to represent a great variety of fractals in a simple way. [4]

Finally, Bidlack explored mappings of both conservative and dissipative systems, besides being one of the first composers to attempt explorations of higher-dimensional maps, as previously described. [2]

## Future Developments

### Small-Scale Chaos

It has been proposed by DiScipio [3] and Truax [11] that "large-scale" applications of chaos to music which construct the song structure itself from the chaotic system do not make musical sense. Truax states: "From a more philosophical or aesthetic point of view, it is not clear than an arbitrary mapping of a non-linear function is inherently more musical than, for instance, a random or stochastic function. The musicality may reside in the musical knowledge of the mapper more than in the source function." [11]

They propose, in opposition, that chaos should be employed to sound synthesis. Specifically, DiScipio proposes the application of chaotic techniques to *granular synthesis*, a technique of sound synthesis where sounds are composed from many "sound grains" being played at different speeds and frequencies. Employing chaos in granular synthesis allows for unpredictable reformulation of these grains, generating endlessly variable sounds. In 2013, an iOS app called "Strange Attractor", based on work by John Mackenzie [7] has been released, which expands on the idea of chaotic sound synthesis.

### Hybrid Techniques

There has been next to no exploration of the possibilities of combining chaotic composition with other algorithmic generation techniques - despite the assertion by many scientists and composers who have researched chaotic composition that perhaps chaotic composition might be better seen as a technique to generate "raw materials" of a certain musical character. [2] It might be fruitful to use chaos as a "generator" of sorts of musical phrases to be combined by more discerning approaches to musical composition, for instance methods based on neural networks.

## Conclusion

We have shown that chaotic systems are a powerful method for the generation of musical phrases - artistically, they are interesting due to the possibility of generating a pattern that is simultaneously regular and near-infinitely variated. We have also explored the philosophical background behind the interest in capturing natural phenomena through music, and shown that chaos allows us to endow the generated music with a "natural" quality, or to produce the aural equivalent of natural phenomena.

We have presented and discussed the most prominent method of converting a chaotic process into music. We have also discussed the possibility of using chaos in the synthesis of sounds, and of combining chaos as an element in more complex algorithmical composition procedures. These are fields which have been woefully underexplored, despite the great potential of chaos.

## References


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## Appendix 1: Figures

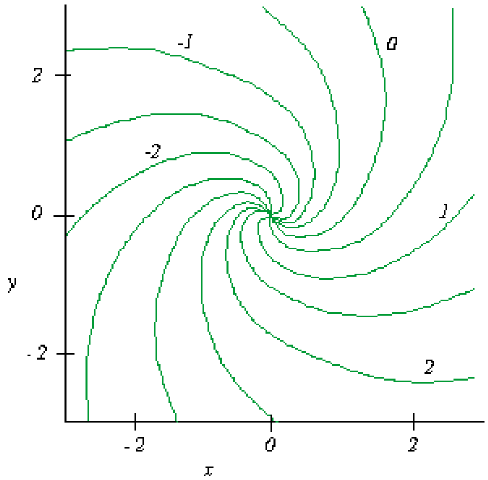


Figure 1: A point attractor.

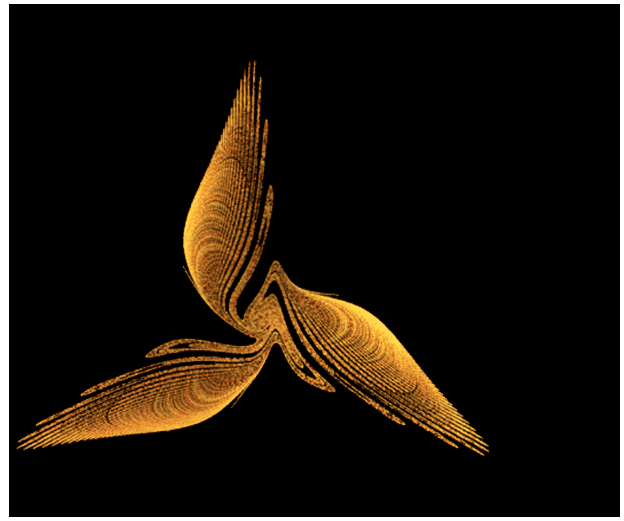

Figure 3: A strange attractor.

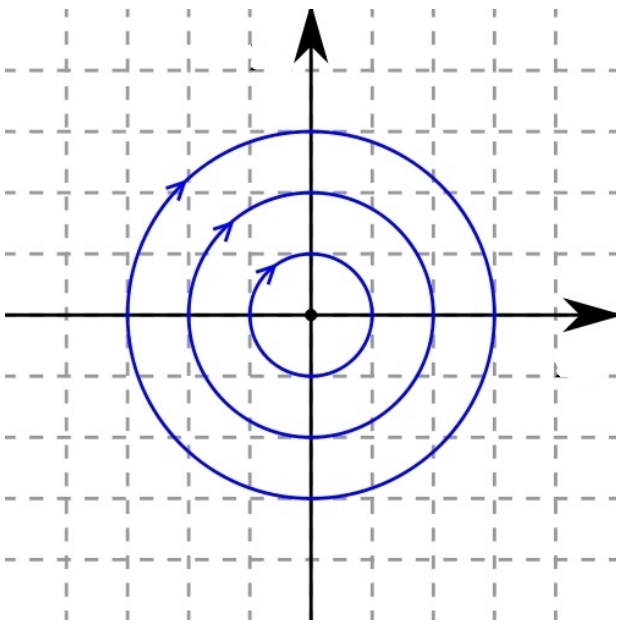

Figure 2: A cycle attractor.

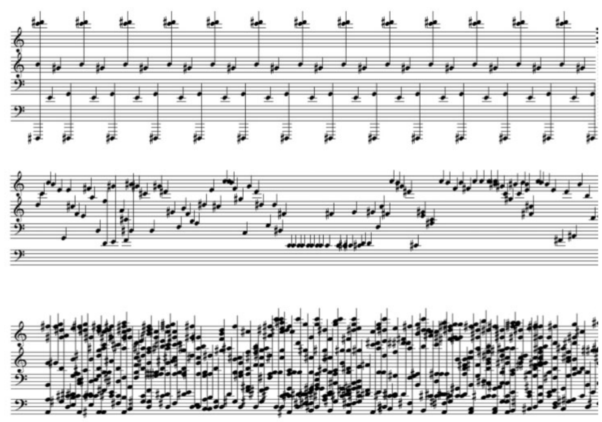

Figure 4: Scores of music generated from attractors. From top to bottom: Music generated from a periodic orbit; Music generated from a small number of chaotic orbits; Music generated from a large number of chaotic orbits.