

# High-Performance Tensor Computations: Where Do We Stand?

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SIAM Conference on Computational Science and Engineering (via Zoom)



High Performance and  
Automatic Computing



UMEÅ UNIVERSITY



HPC2N

# About me (and tensors)

- ▶ Taxonomy of contractions: Can you GEMM? E. Di Napoli, D. Traver-Fabregat, P.B.  
*"Towards an Efficient Use of the BLAS Library for Multilinear Tensor Contractions", AMC 235, 2014*
- ▶ Performance prediction E. Peise, P.B.  
*"On the Performance Prediction of BLAS-based Tensor Contractions", PMBS, SC'14*
- ▶ Density Functional Theory: FLAPW methods E. Di Napoli, E. Peise, P.B.  
*"High-Performance Generation of the Hamiltonian and Overlap Matrices in FLAPW Methods", CPC 2017*
- ▶ High-performance kernels P. Springer, P.B.  
*"TTC: A high-performance Compiler for Tensor Transpositions", ACM TOMS 44(2), 2017* + J. Hammond  
*"Design of a High-Performance GEMM-like Tensor-Tensor Multiplication", ACM TOMS 44(3), 2018*  
*"Spin Summations: A High-Performance Perspective", ACM TOMS 45(1), 2019* + D. Matthews
- ▶ High-intensity kernels C. Psarras, L. Karsson, P.B.  
*"Concurrent Alternating Least Squares for multiple simultaneous Canonical Polyadic Decompositions", 2020*

# Matrices vs. Tensors

Historical overview

# Linear Algebra Libraries: 1970s

*"Basic Linear Algebra Subprograms for FORTRAN usage", ACM TOMS, 1979*

BLAS-1

# Linear Algebra Libraries: 1980s

BLAS-2: Mat-vec ops, ACM TOMS 1988.

BLAS-3: mat-mat ops, ACM TOMS 1990

BLAS-1, BLAS-2, BLAS-3

# Linear Algebra Libraries: 1990s

Solvers & eigensolvers, 1992

LAPACK

BLAS-1, BLAS-2, BLAS-3

# Linear Algebra Libraries: 1990s

Distributed Memory, 1995, 1997

ScaLAPACK, PLAPACK, ...

LAPACK

BLAS-1, BLAS-2, BLAS-3

# Linear Algebra Libraries: 1990s

Dense & Sparse, 1997

PETSc, ...

ScaLAPACK, PLAPACK, ...

LAPACK

BLAS-1, BLAS-2, BLAS-3



# Linear Algebra Libraries

and then more!

PETSc, Trilinos, ...

ScaLAPACK, PLAPACK, Elemental, ...

LAPACK, Plasma, SuperMatrix, Magma, ...

BLAS-1, BLAS-2, BLAS-3, ATLAS, BTO-BLAS, BLIS, ...

# (Dense) Linear Algebra Libraries

## Salient features

- ▶ Community effort. Standardized interface
- ▶ Careful organization: support routines, linear-systems, eigen-decompositions
- ▶ Clear layering: functionality, parallelism

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But

- ▶ Rigid interface
- ▶ Black-box nature
- ▶ Often sub-optimal at small scale

$$K_k := P_k^b H^T (H P_k^b H^T + R)^{-1}; \quad x_k^a := x_k^b + K_k (z_k - H x_k^b); \quad P_k^a := (I - K_k H) P_k^b$$

$$\begin{cases} C_{\dagger} := P C P^T + Q \\ K := C_{\dagger} H^T (H C_{\dagger} H^T)^{-1} \end{cases}$$

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MUL   ADD   MOV  
 MOVAPD  
 VFMADDPD   ...

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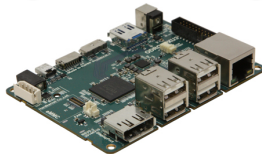
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...  BLAS  LAPACK  ...



MUL ADD MOV

MOVAPD

VFMADDPD ...

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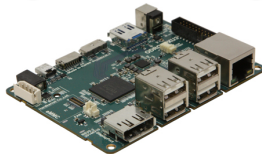
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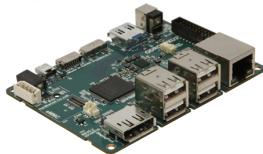
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**LINEAR ALGEBRA  
MAPPING PROBLEM  
("LAMP")**

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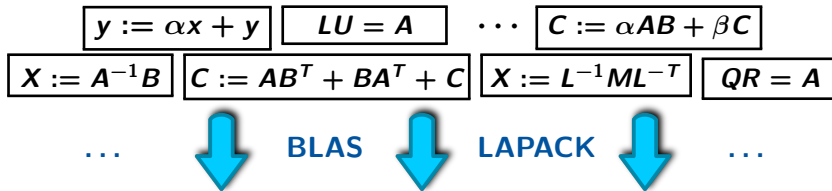
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**LINEAR ALGEBRA  
MAPPING PROBLEM  
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C. Psarras, H. Barthels, "The Linear Algebra Mapping Problem. Current state of linear algebra languages and libraries". [arXiv:1911.09421]

H. Barthels, C. Psarras, "Linnea: Automatic Generation of Efficient Linear Algebra Programs", ACM TOMS, 2021. [arXiv:1912.12924]

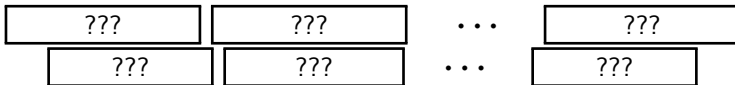
# Tensors

Tensor App #1

Tensor App #2

...

Tensor App #N



BLAS  
LAPACK



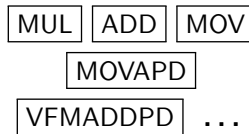
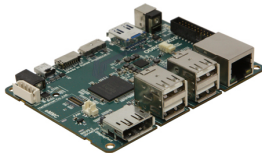
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...



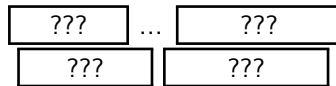
???



but...

Comp. Physics  
Comp. Chemistry

Data Science  
Machine Learning

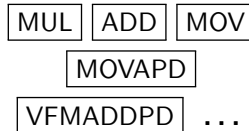
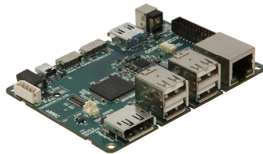


BLAS  
LAPACK

???

...

???



# Tensor computations

- ▶ (At least) Two separate worlds

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<sup>1</sup>With notable differences.

# Tensor computations

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- ▶ Computational physics / chemistry

Tensor = Multi-linear operator

Contractions = Generalization of matrix-matrix product

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# Tensor computations

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- ▶ Data science

- Tensor = Collection of data

- Decompositions = Generalization of matrix factorizations<sup>1</sup>

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# Tensor computations

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  - ▶ Computational physics / chemistry
    - Tensor = Multi-linear operator
    - Contractions = Generalization of matrix-matrix product
  - ▶ Data science
    - Tensor = Collection of data
    - Decompositions = Generalization of matrix factorizations<sup>1</sup>
- ▶ Terminology and notation vary (and conflict) even within one world
- ▶ Very few software efforts cut across the boundary

---

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## MS14 (today) & MS79 (tomorrow)

- ▶ (At least) Two separate worlds
  - ▶ Computational physics / chemistry
    - Quantum Chemistry: Ed Valeev, Devin Matthews, Edgar Solomonik
    - Quantum Physics: Pan Zhang, Lei Wang, Miles Stoudenmire
  - ▶ Data science
    - Furong Huang, Hanie Sedghi, Vagelis Papalexakis
- ▶ Notation: Miles Stoudenmire
- ▶ Software: Edgar Solomonik

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## Tensors, presently

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- ▶ No community effort!  
A jungle of independent libraries and packages, in a variety of languages  
Massive redundancy: replication of effort, low performance
- ▶ Application-driven development:  
Publications scattered among different fields  
Re-invention of the wheel

## Matlab and R packages with support for CP decomposition (subset)

- ▶ **Tensor Toolbox** by Bader, Kolda, & others  
<https://www.tensortoolbox.org/>
- ▶ **Tensorlab** by Vervliet, Debals, Sorber, Van Barel, & De Lathauwer  
<https://www.tensorlab.net/index.html>
- ▶ **The N-way Toolbox** by Bro & Andersson  
<http://www.models.life.ku.dk/nwaytoolbox>
- ▶ **TensorBox** by Phan, Tichavsky, & Cichocki  
<https://github.com/phananh Huy/TensorBox>
- ▶ **Tensor Package** by Comon & others  
<http://www.gipsa-lab.fr/~pierre.comon/TensorPackage/tensorPackage.html>
  
- ▶ **multiway** by Helwig  
<https://cran.r-project.org/package=multiway>
- ▶ **ThreeWay** by Giordani, Kiers, & Del Ferraro  
<https://cran.r-project.org/package=ThreeWay>
- ▶ **rTensor** by Li, Bien, & Wells  
<https://cran.r-project.org/package=rTensor>

## C/C++ packages with support for CP decomposition (subset)

- ▶ **Genten** by SANDIA (Phipps)  
<https://gitlab.com/tensors/genten>
- ▶ **SPLATT** by Smith & Karypis  
<https://github.com/ShadenSmith/splatt>
- ▶ **ParTI!** by Li, Ma, & Vuduc  
<https://github.com/hpcgarage/ParTI>
- ▶ **Cyclops** by Solomonik & others  
<https://github.com/cyclops-community>

And then there's Python, Fortran, ...

# Representative operations – building blocks candidates

## Data layout operations

- ▶ Reshape
- ▶ Permute / transpose
- ▶ Sort (sparse)
- ▶ Convert data layout
- ▶ Partition
- ▶ Distribute
- ▶ ...

## Arithmetic operations

- ▶ Add, subtract, scale
- ▶ Inner product
- ▶ Norms
- ▶ Element-wise operations
- ▶ Tensor-times-vector (TTV)
- ▶ Tensor-times-matrix (TTM)
- ▶ MTTKRP
- ▶ Contractions
- ▶ ...

## Decompositions

- ▶ CP  
(CANDECOMP/PARAFAC)
- ▶ Tucker
- ▶ INDSCAL
- ▶ PARAFAC2
- ▶ CANDELINC
- ▶ DEDICOM
- ▶ PARATUCK2
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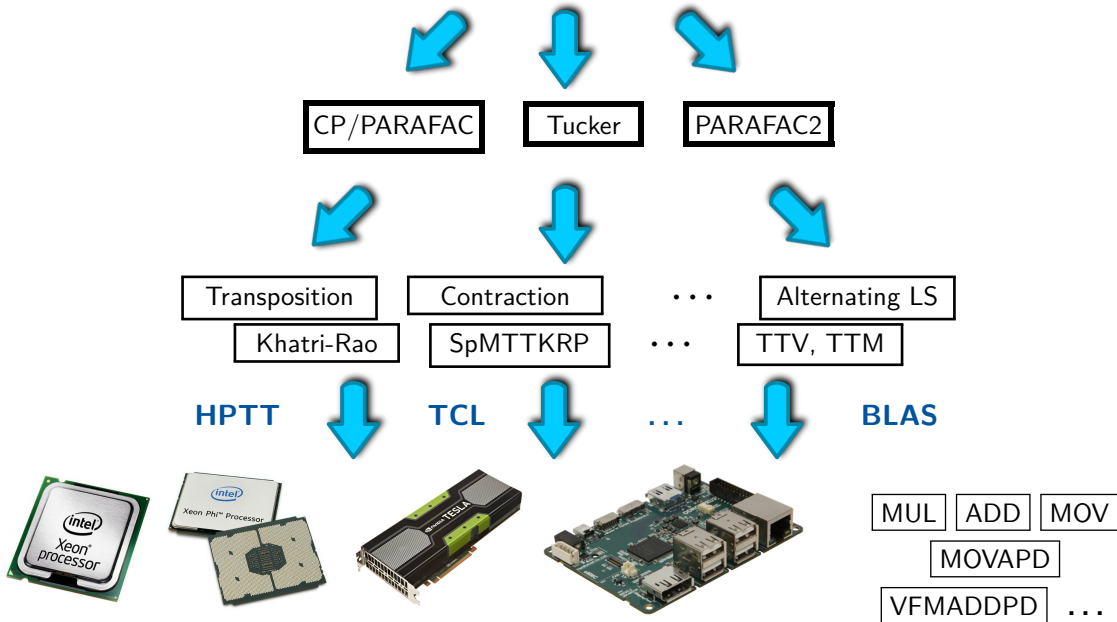
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- ▶ ...

In setting up a library, where to draw the boundaries?



# 2-level "solution"

## Chromatography-MS



# Algorithms for CP (PARAFAC) decomposition

Hence all the different libraries

## ▶ Algebraic algorithms

- ▶ Generalized Rank Annihilation Method
- ▶ Direct TriLinear Decomposition
- ▶ The “algebraic algorithm”  
by Domanov and De Lathauwer
- ▶ The “simpler algorithm”  
by Pimentel-Alarcón
- ▶ ...

## ▶ Alternating optimization algorithms

- ▶ Alternating Least Squares
- ▶ Fast ALS
- ▶ Hierarchical ALS
- ▶ Regularized ALS
- ▶ ...

## ▶ All-at-once optimization algorithms

- ▶ Gradient descent
- ▶ (Damped) Gauss–Newton
- ▶ Nonlinear CG, GMRES
- ▶ Quasi-Newton (e.g., L-BFGS)
- ▶ ...

## ▶ Enhancements

- ▶ Line search
- ▶ Compression
- ▶ Randomization
- ▶ Transient constraints
- ▶ ...

Also ...

# Also ... one vs. many instances

## Coupled-Cluster methods

$$\tilde{\tau}_{ij}^{ab} = t_{ij}^{ab} + \frac{1}{2} P_b^a P_j^i t_i^a t_j^b,$$

$$\tilde{F}_e^m = f_e^m + \sum_{fn} v_{ef}^{mn} t_n^f,$$

$$\tilde{F}_e^a = (1 - \delta_{ae}) f_e^a - \sum_m \tilde{F}_e^m t_m^a - \frac{1}{2} \sum_{mnf} v_{ef}^{mn} t_{mn}^{af} + \sum_{fn} v_{ef}^{an} t_n^f,$$

$$\tilde{F}_i^m = (1 - \delta_{mi}) f_i^m + \sum_e \tilde{F}_e^m t_i^e + \frac{1}{2} \sum_{nef} v_{ef}^{mn} t_{in}^{ef} + \sum_{fn} v_{if}^{mn} t_n^f,$$

$$\tilde{W}_{ei}^{mn} = v_{ei}^{mn} + \sum_f v_{ef}^{mn} t_i^f,$$

$$\tilde{W}_{ij}^{mn} = v_{ij}^{mn} + P_j^i \sum_e v_{ie}^{mn} t_j^e + \frac{1}{2} \sum_{ef} v_{ef}^{mn} \tau_{ij}^{ef},$$

$$\tilde{W}_{ie}^{am} = v_{ie}^{am} - \sum_n \tilde{W}_{ei}^{mn} t_n^a + \sum_f v_{ef}^{ma} t_i^f + \frac{1}{2} \sum_{nf} v_{ef}^{mn} t_{in}^{af},$$

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$$z_i^a = f_i^a - \sum_m \tilde{F}_i^m t_m^a + \sum_e f_e^a t_i^e + \sum_{em} v_{ei}^{ma} t_m^e + \sum_{em} v_{im}^{ae} \tilde{F}_e^m + \frac{1}{2} \sum_{efm} v_{ef}^{am} t_{im}^{ef},$$

$$z_{ij}^{ab} = v_{ij}^{ab} + P_j^i \sum_e v_{ie}^{ab} t_j^e + P_b^a P_j^i \sum_{me} \tilde{W}_{ie}^{am} t_{mj}^{eb} - P_b^a \sum_m \tilde{W}_{ij}^{am} t_m^b + P_j^i \sum_e v_{ie}^{ab} t_j^e,$$

credits to D. Matthews, E. Solomonik, J. Stanton, and J. Gauss

## Also ... one vs. many instances

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$$z_{ij}^{ab} = v_{ij}^{ab} + P_j^i \sum_e v_{ie}^{ab} t_j^e + P_b^a P_j^i \sum_{me} \tilde{W}_{ie}^{am} t_{mj}^{eb} - P_b^a \sum_m \tilde{W}_{ij}^{am} t_m^b + P$$

credits to D. Matthews, E. Solomonik, J. Stanton, and J. Gauss

### Finite Element 3D diffusion operator

```
TE.BeginMultiKernelLaunch();
TE("T2_e_i1_i2_k3 = B_k3_i3 X_e_i1_i2_i3", T2, B, X);
TE("T1_e_i1_k2_k3 = B_k2_i2 T2_e_i1_i2_k3", T1, B, T2);
TE("U1_e_k1_k2_k3 = G_k1_i1 T1_e_i1_k2_k3", U1, G, T1);
TE("T1_e_i1_k2_k3 = G_k2_i2 T2_e_i1_i2_k3", T1, G, T2);
TE("U2_e_k1_k2_k3 = B_k1_i1 T1_e_i1_k2_k3", U2, B, T1);
TE("T2_e_i1_i2_k3 = G_k3_i3 X_e_i1_i2_i3", T2, G, X);
TE("T1_e_i1_k2_k3 = B_k2_i2 T2_e_i1_i2_k3", T1, B, T2);
TE("U3_e_k1_k2_k3 = B_k1_i1 T1_e_i1_k2_k3", U3, B, T1);
TE("Z_m_e_k1_k2_k3 = U_n_e_k1_k2_k3 D_e_m_n_k1_k2_k3", Z, U,
TE("T1_e_i3_k1_k2 = B_k3_i3 Z1_e_k1_k2_k3", T1, B, Z1);
TE("T2_e_i2_i3_k1 = B_k2_i2 T1_e_i3_k1_k2", T2, B, T1);
TE("Y_e_i1_i2_i3 = G_k1_i1 T2_e_i2_i3_k1", Y, G, T2);
TE("T1_e_i3_k1_k2 = B_k3_i3 Z2_e_k1_k2_k3", T1, B, Z2);
TE("T2_e_i2_i3_k1 = G_k2_i2 T1_e_i3_k1_k2", T2, G, T1);
TE("Y_e_i1_i2_i3 += B_k1_i1 T2_e_i2_i3_k1", Y, B, T2);
TE("T1_e_i3_k1_k2 = G_k3_i3 Z3_e_k1_k2_k3", T1, G, Z3);
TE("T2_e_i2_i3_k1 = B_k2_i2 T1_e_i3_k1_k2", T2, B, T1);
TE("Y_e_i1_i2_i3 += B_k1_i1 T2_e_i2_i3_k1", Y, B, T2);
TE.EndMultiKernelLaunch();
```

credits to A. Fisher – <https://github.com/LLNL/acrotensor>

# Same in data science: Gas Chromatography

## Workflow

- ...
4. **Fit model or rank  $k \in [1, \dots, 15]$** , if needed, add non-negativity constraints  
Tensor decompositions: PARAFAC — PARAFAC2 — TUCKER
  5. Determine whether or not one of the models is “right”
    - ▶ 😊: Determine which of the components represent chemical information
    - ▶ 😞: Start over; add/change constraints, change model

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Computation of each individual model: **bandwidth bound!**

Hence: “*Concurrent Alternating Least Squares for multiple simultaneous Canonical Polyadic Decompositions*”, with C. Psarras, L. Larsson. (Submitted).

## Summary

	<b>Matrices</b>	<b>Tensors</b>
<b>Driver</b>	performance, HW	applications
<b>Community effort</b>	BLAST/LAPACK/...	group by group
<b>Industry</b>	wide support	not much
<b>Standardization</b>	interface, ...	“pointless”
<b>Preferred outlet</b>	ACM TOMS	—
<b>Language support</b>	plenty	language by language
<b>Automation</b>	plenty	TCE (2001), but then?



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Still a long way to maturity! — Thank you for the attention.