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Towards Automated Load Balancing via Spectrum Slicing for FEAST-like Solvers

30.11.2016 | Jan Winkelmann, Edoardo Di Napoli

The Hermitian Interior Eigenvalue Problem

- Let $H \in \mathbb{C}^{n \times n}$ be Hermitian
- Find all (λ, v) with $\lambda \in [a, b]$, such that

$$Hv = \lambda v$$

- All solvers are sensitive to the *spectrum* $\{\lambda \mid Hv = \lambda v\}$
- Knowledge about the spectrum may increase performance
- Today: Load balancing for FEAST-like eigensolvers

Our goal is to combine

1. The ability to design filters with specific properties
2. Cheaply available information on the spectrum

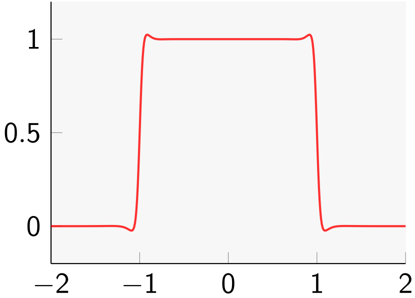
In order to achieve good parallel performance on FEAST-like solvers.

FEAST: A Rationally Filtered Interior Eigensolver

FEAST(Matrix A, Approx Y)

while $||Y^*AY - \Lambda|| < \texttt{tol}$ **do**
 $U \leftarrow f(A)Y$
 $U \leftarrow QR(U)$
 $B \leftarrow U^*AU$
 Solve: $\Lambda = W^*BW$
 $Y \leftarrow UW$
return (Λ, Y)

$$f(A) = \sum_{i=1}^{p} \alpha_i (Iz_i - A)^{-1}$$



Levels of Parallelism in FEAST

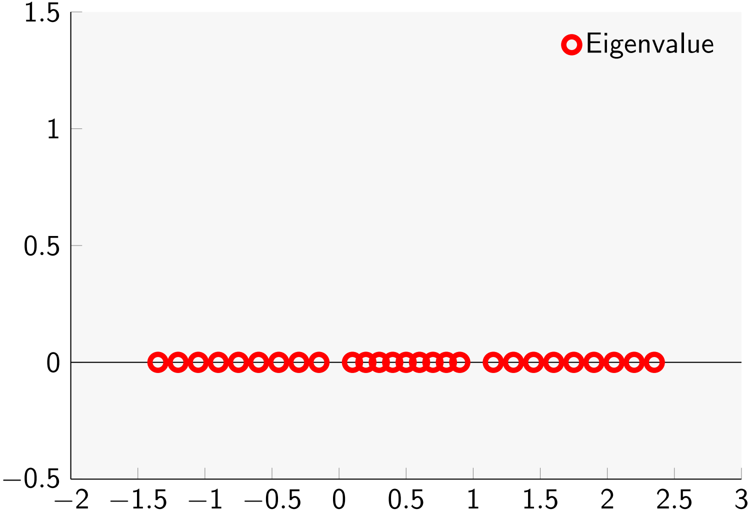
1. Partition the search interval into multiple, indendent intervals
2. p Linear System Solves per iteration

$$U := f(A)Y = \sum_{i=1}^{p} \alpha_i (Iz_i - A)^{-1} Y$$

3. Linear System Solve

$$(A - Iz_i)U = \alpha_i Y$$

State of the Art: Approximate Eigenvalue Counts



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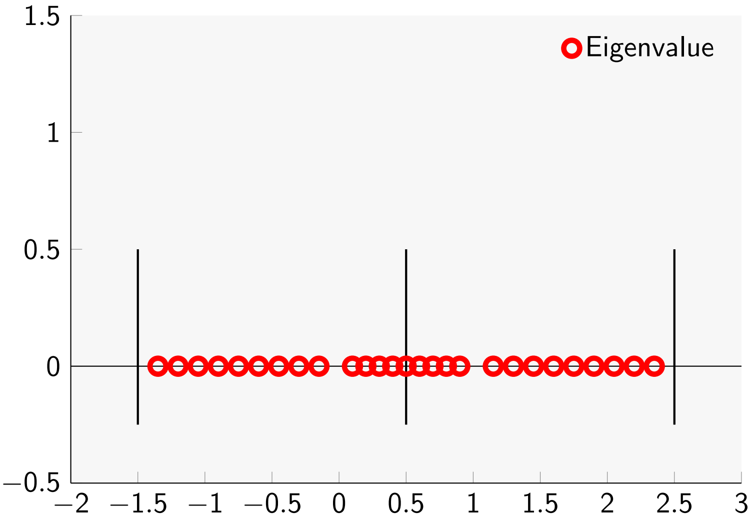

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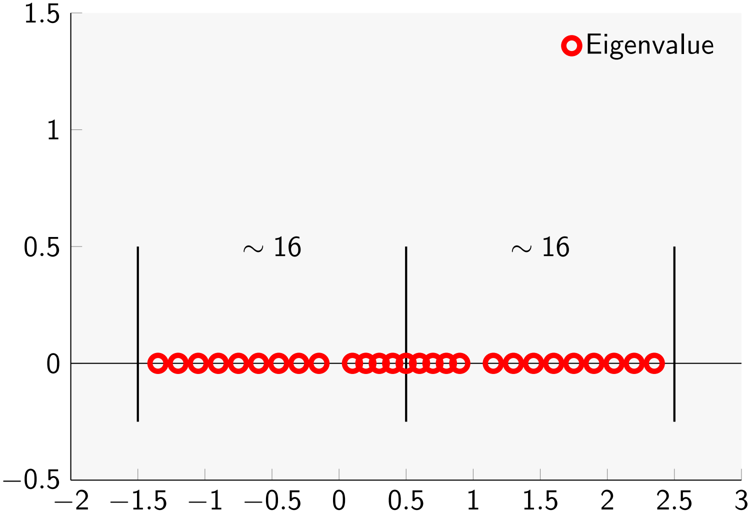

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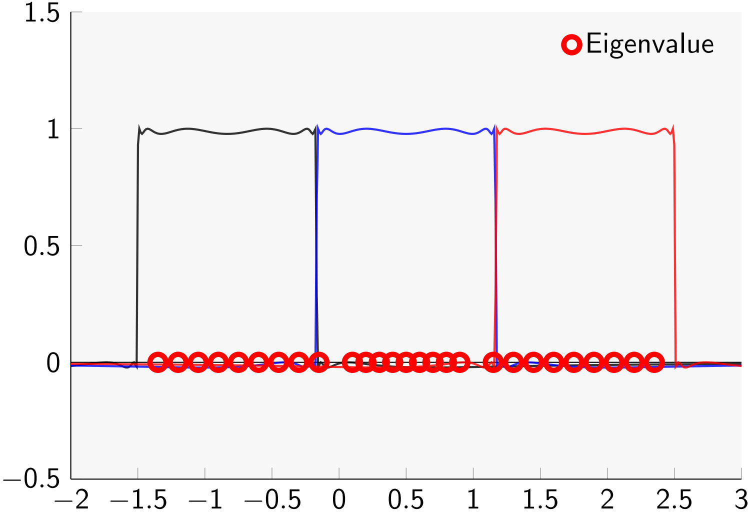

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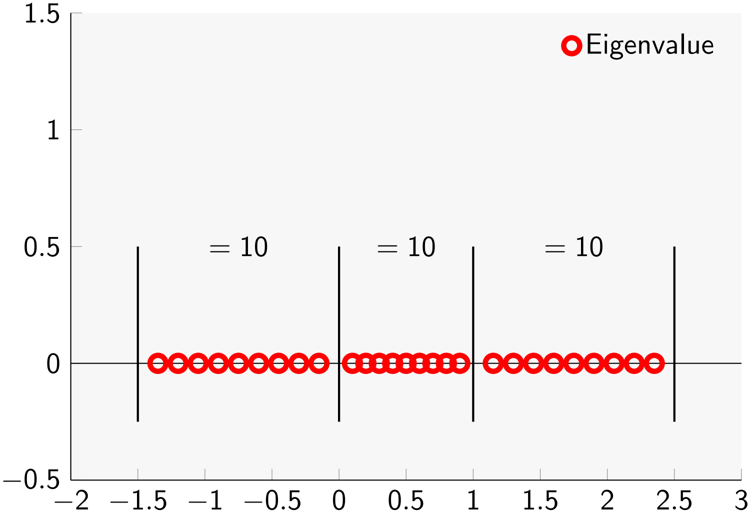


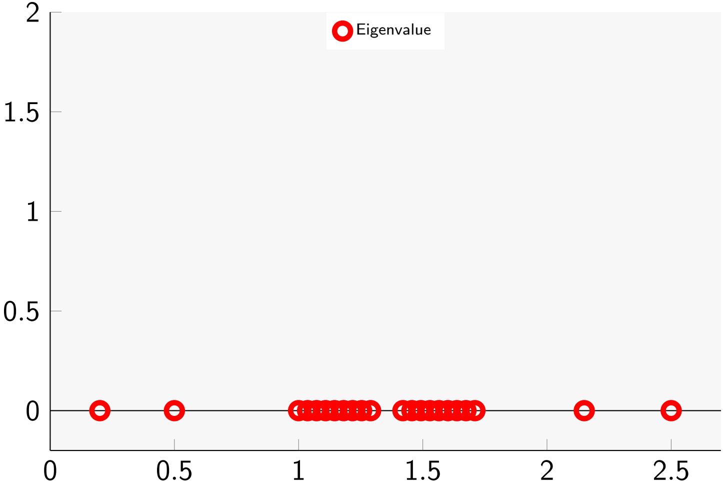
NLLS filters: Optimized filters for FEAST

Non-Linear Least-Squares optimized filters:

$$\min_{\alpha_i, z_i \; 1 \leq i \leq n} \int_{-\infty}^{\infty} w(t) \left| h(t) - \sum_{i=1}^{n} \frac{\alpha_i}{t - z_i} \right|^2 dt$$

- Good results when compared to existing filters
- Ability to 'design' filters for special use-case
- Forthcoming publication in beginning of 2017

Future Work: Load Balancing with NNLS Filters and DoS



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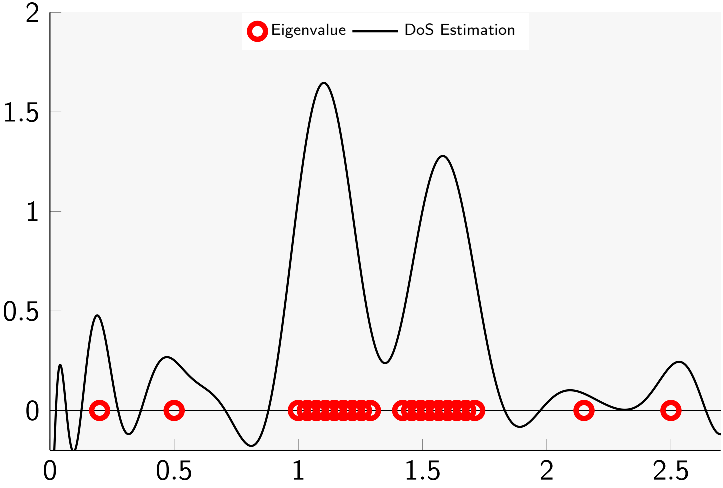

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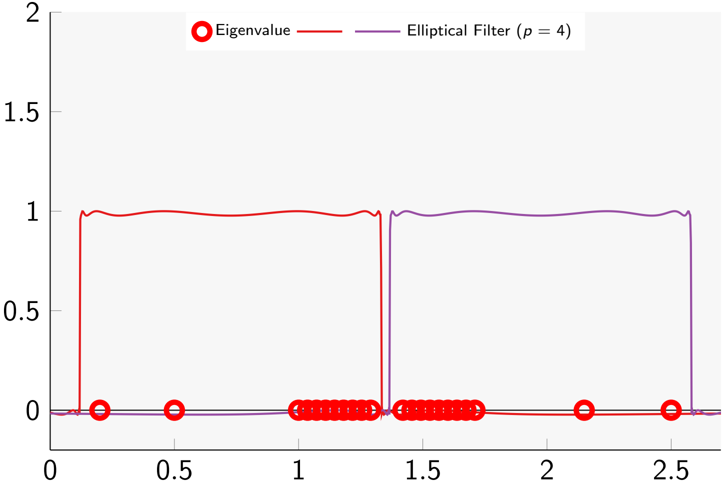

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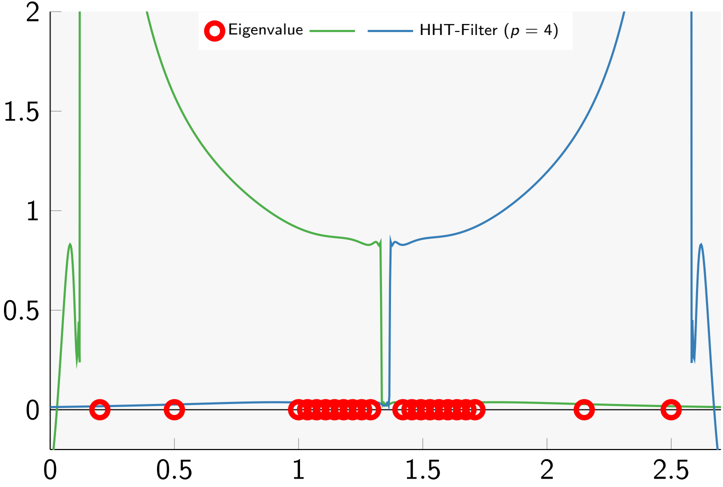

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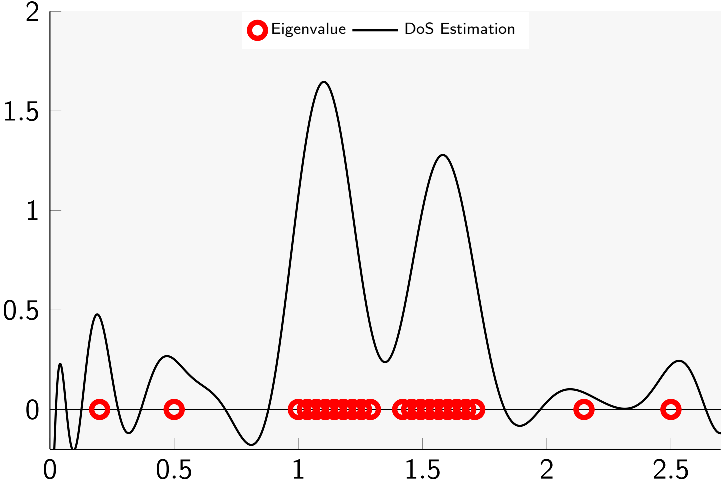

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Opportunities for Collaboration

Cheap Information on the Spectrum

- Given Matrix A, sparse/dense, (non-)Hermitian
- Obtain information on the spectrum of A around $[a, b]$
- As cheaply as possible (Trace estimation, Lanczos)

Eigenproblems with Difficulties in Load Balancing

- Preferably Hermitian problems
- Suspected potential for better Load Balancing
- Ideally with FEAST-like solvers

The End

Thank you for your attention

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Future Work: Load Balancing with Krylov-Solvers

The Linear System Solve:

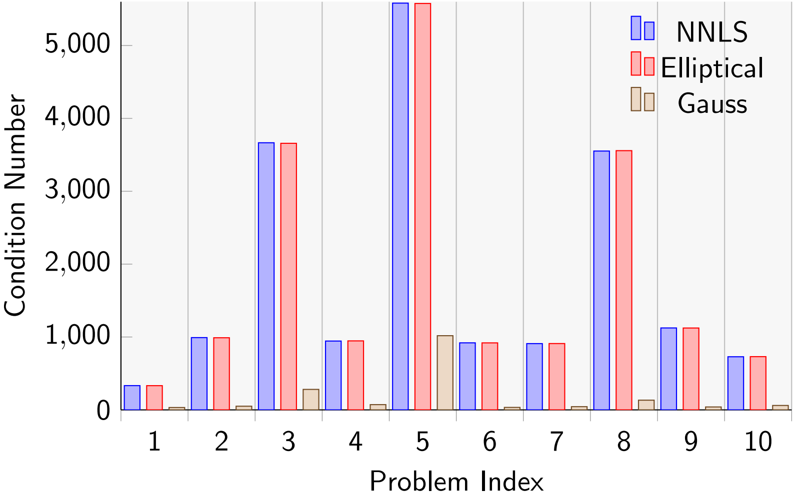
$$(A - Iz_i)U = \alpha_i Y \qquad \forall 1 \leq i \leq p$$

- For large p $\min_{1 \leq i \leq p} |\Im(z_i)|$ becomes very small
- Then $(A - Iz_i)$ may become nearly singular
- Workload imbalance with Krylov-based solvers

Another aspect of load balancing

- We can influence the imaginary part of z_i
- Reduces Load Imbalance (at cost of convergence speed)

Condition Number of $(A - Iz_i)$



Performance Profile